

# Comparison of Dual-Polarization Radar Estimators of Rain

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# Overview

- ◆ Motivation and Background Info
  - Motivation behind improved rainfall estimates.
  - Past studies
- ◆ Relations between polarimetric variables and rain amounts.
  - Simulated drop size distributions (DSD)
- ◆ Compare different rain rate ( $R$ ) and liquid water content ( $M$ ) measurements.
  - Simulation
  - Errors
  - Comparison with rain gauges
- ◆ Conclusions

# Motivation:

- ◆ Why do we need improved accuracy in rainfall estimation?
  - Safety: Flooding (high rainfall rates).
  - Improved initialization of hydrological models.
  - Agriculture: Highlight areas where crop shortfalls or flooding is likely.

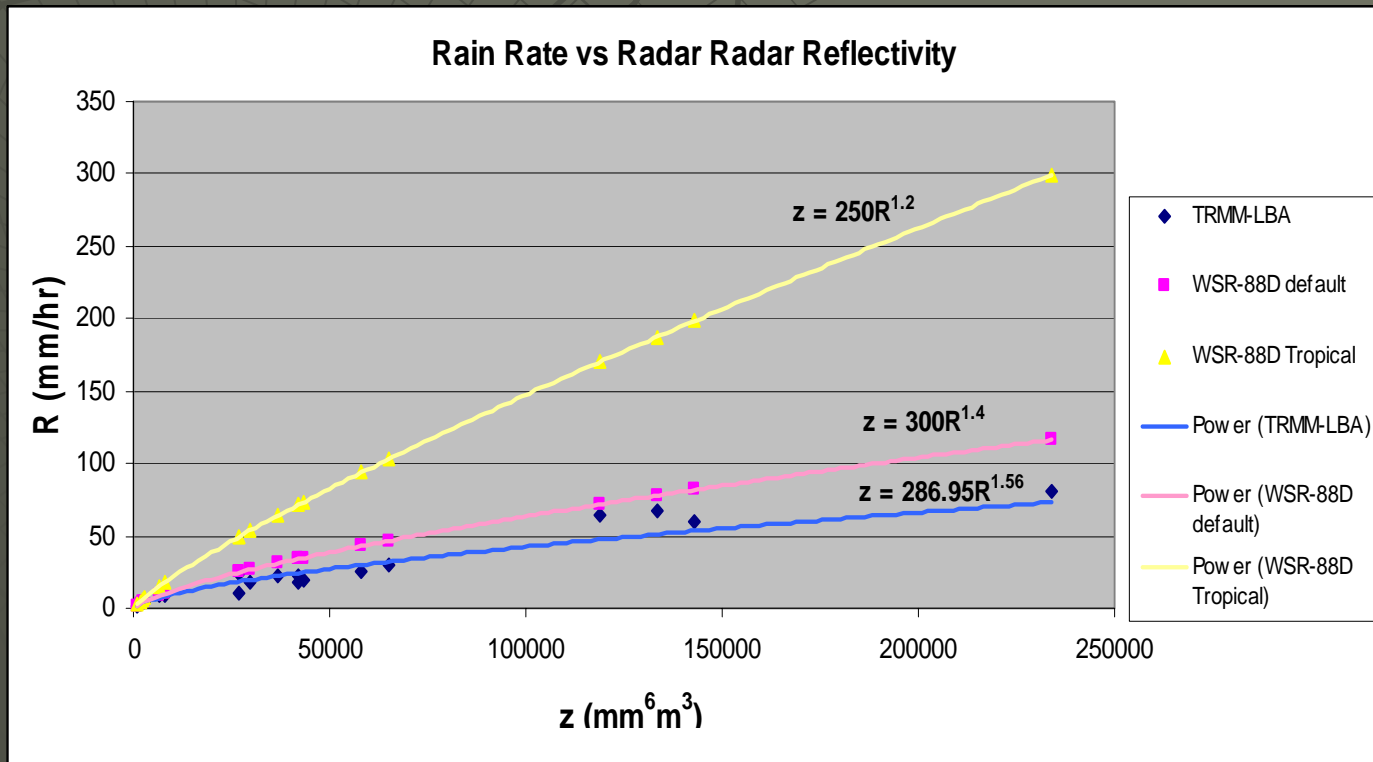


# How is Rain Fall Measured?

- ◆ Rain gages:
  - Useful, but only serve as isolated point measurements.
  - Gauge measurements are often assumed to represent the average rainfall rate over large areas.
- ◆ Therefore, it is necessary to identify alternative rainfall measurement techniques.

# How is Rain Fall Measured?

- ◆ WSR-88D:
  - Several studies have attempted to construct correlations between the reflectivity factor (Z) and rainfall rate (R).
  - Useful for rainfall estimates over large areas for long periods of time.
  - Generally useless for hydrologic problems requiring good temporal and spatial resolution.



# Motivation

- ◆ WSR-88D:
  - ◆ Default:  $Z = 300R^{1.4}$
  - ◆ Tropical:  $Z = 250R^{1.2}$ 
    - ◆ Jameson (1991): *One of the consistently worse parameters (largest  $\varepsilon$  where  $\varepsilon$  is standard deviation) for estimating nearly instantaneous, point rainfall rates appears to be the reflectivity factor ( $Z$ )*
- ◆ How can Polarimetric variables assist in precipitation estimation?

# Background

- ◆ Polarimetric variables are not immune to errors.
  - Methods involving radar reflectivity require knowledge of absolute powers of radar system; therefore they are prone to calibration issues
  - Although  $K_{dp}$  is independent of radar calibrations it only has superior accuracy at high rain rates.

# Errors in Radar Rainfall Estimates

- ◆ Variations of drop size distributions (DSD).
  - The uncertainty in the median drop diameter  $D_0$  is the main source of DSD induced errors in  $R(K_{DP})$ .
    - ◆ Solution: Find a factor that could compensate for the effects of the variations of  $D_0$  such that  $R(K_{DP})$  is independent of DSDs.

- ◆ Jameson (1991): Mass-weighted mean axis ratio is such a factor, which can be measured with  $Z_{DR}$ 
  - Given axis ratio ( $a/b$ ) use results from Pruppacher and Beard (1970) to find  $D$ :

$$a/b = 1.03 - 0.62D, \quad D \geq 0.1cm$$

- ◆ Using best-fit relation Jameson (1991) developed the following rain rate relation:

$$R\left(\frac{Z_h}{Z_v}, K_{DP}\right) = 6.242 K_{DP}^{0.975} \left[ 1 - \left(\frac{Z_h}{Z_v}\right)^{-3/7} \right]^{-0.975}$$

where

R: rain rate in millimeters per hour

$K_{DP}$ : Specific differential phase in degrees per kilometer

$Z_h$   $Z_v$ : Horizontal and vertical radar reflectivity, respectively ( $\text{mm}^6\text{m}^{-3}$ )

Strength of relation can be measured by a normalized correlation coefficient between the distributions  $X(D)$  and  $R(D)$  over drop size  $D$  (where  $X$  represents radar parameter and  $R$  is rain rate). The smaller the correlation coefficient, the less  $X$  and  $R$  are related

- ◆ In order to incorporate  $Z_{dr}$  into their rain rate equation, Ryzhkov and Zrnic (1995) suggest a power-law relation of the same form, i.e.

$$cK_{DP}^a Z_{DR}^b$$

- ◆ First step in finding the desired power-law relation involved simulations of DSD variations using a gamma distribution:

## ◆ Gamma Distribution:

$$N(D) = N_0 D^\mu \exp(-AD)$$

$$N(D) = N_0 D^\mu \exp\left[-(3.67 + \mu) \frac{D}{D_0}\right]$$

- ◆ Where  $N_0$ : Concentration parameter
- ◆  $\mu$ : Shape parameter
- ◆ A: Slope parameter
- ◆ D: equivolume drop diameter (max = 8 mm)
- ◆  $D_0$ : median drop diameter

- ◆ DSDs were determined by varying the parameters within the following intervals:

$$10^{3.2-\mu} \exp(2.8\mu) < N_0 (m^{-3} mm^{-1-\mu}) < 10^{4.5-\mu} \exp(3.57\mu)$$

Ulbrich, 1983

$$N(D) = N_0 D^\mu \exp\left[-(3.67 + \mu) \frac{D}{D_0}\right]$$

$$R = 0.6 \times 10^4 \pi \rho_w \int v(D) D^3 N(D) dD$$

Where D: Mass equivalent drop diameter (cm)

$\rho_w$ : density of water

$N(D)dD$ : concentration of drops in size interval D to D+dD

$v(D)$ : terminal fall speed in still air (cm/s)

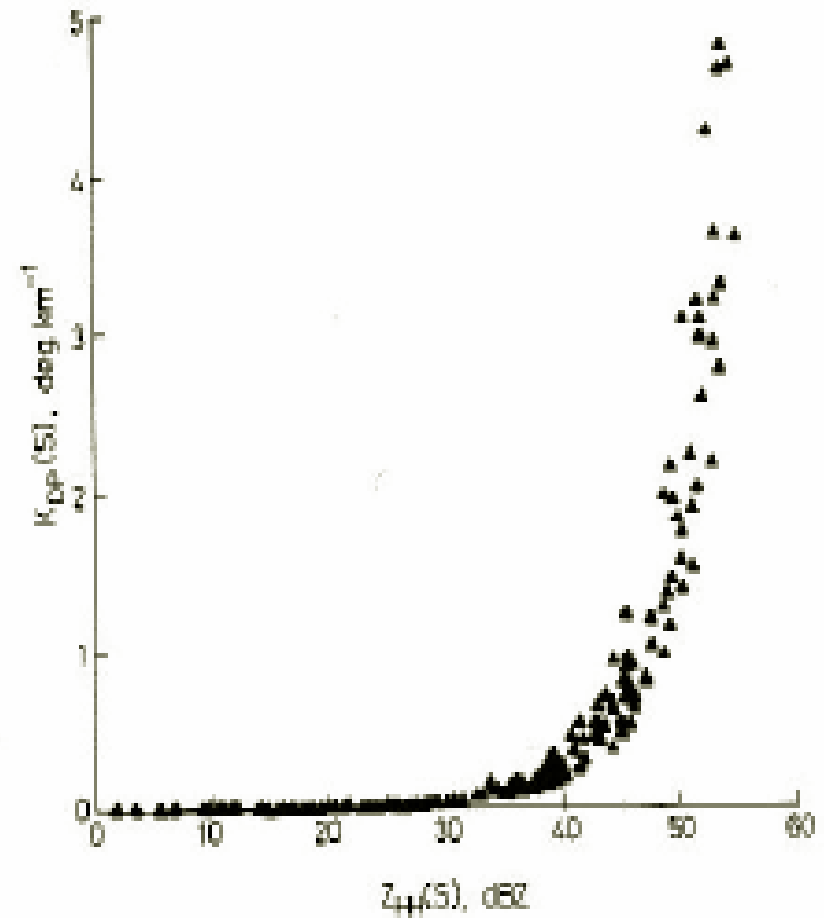
- ◆ Calculations of  $K_{DP}$ ,  $Z_{DR}$ ,  $R$ , and liquid water content ( $M$ ) were made using a wavelength of 10.97cm (wavelength of NSSL's polarimetric radar)
- ◆ Using a standard nonlinear regression technique, a best fit relation for  $R(K_{DP}, Z_{DR})$  and  $M(K_{DP}, Z_{DR})$  was obtained:

$$R = 52.0 K_{DP}^{0.96} Z_{DR}^{-0.447}$$

$$M = 3.11 K_{DP}^{0.918} Z_{DR}^{-0.764}$$

- ◆  $R$ (mm/hr)       $M$ (g/cm<sup>3</sup>)       $K_{DP}$  (deg/km)  
 $Z_{DR}$ (dB)

- ◆ 64% of simulated DSDs corresponded to light precipitation ( $R < 20$  mm/hr).
- ◆  $K_{DP}$  too small at these rain rates to be measured accurately.
- ◆ Typical standard error of  $K_{DP}$  is approx  $\frac{1}{4}$  deg/km.



**Fig. 2** Computations of  $K_{DP}$  against  $Z_{DP}$  for ablate raindrops assuming a gamma size distribution

Each data point refers to a particular value of the triplet of gamma parameters ( $N_0$ ,  $D_0$ ,  $m$ ). The scatter of the data reflects a wide variety of rainfall types  
 a S-band                      b C-band

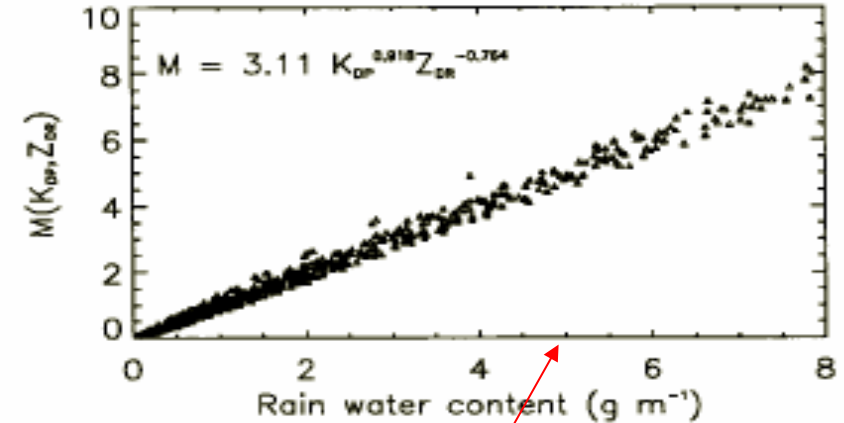
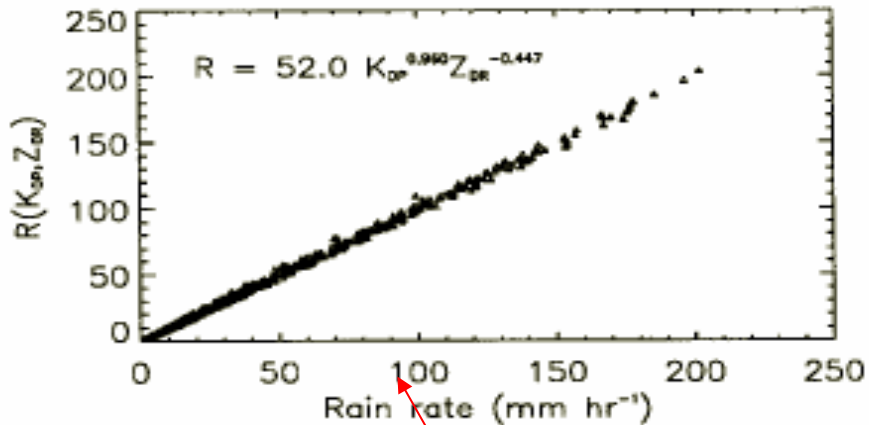
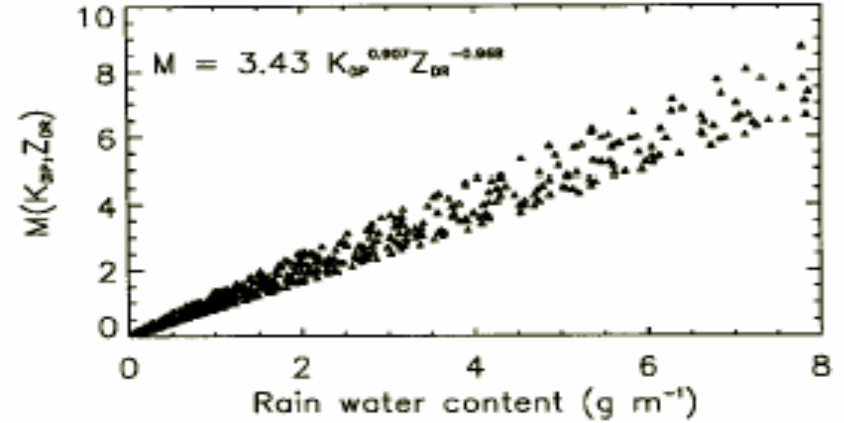
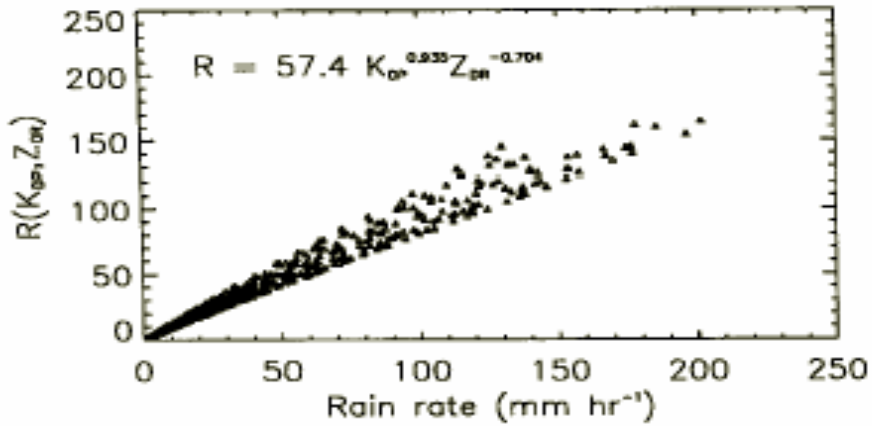
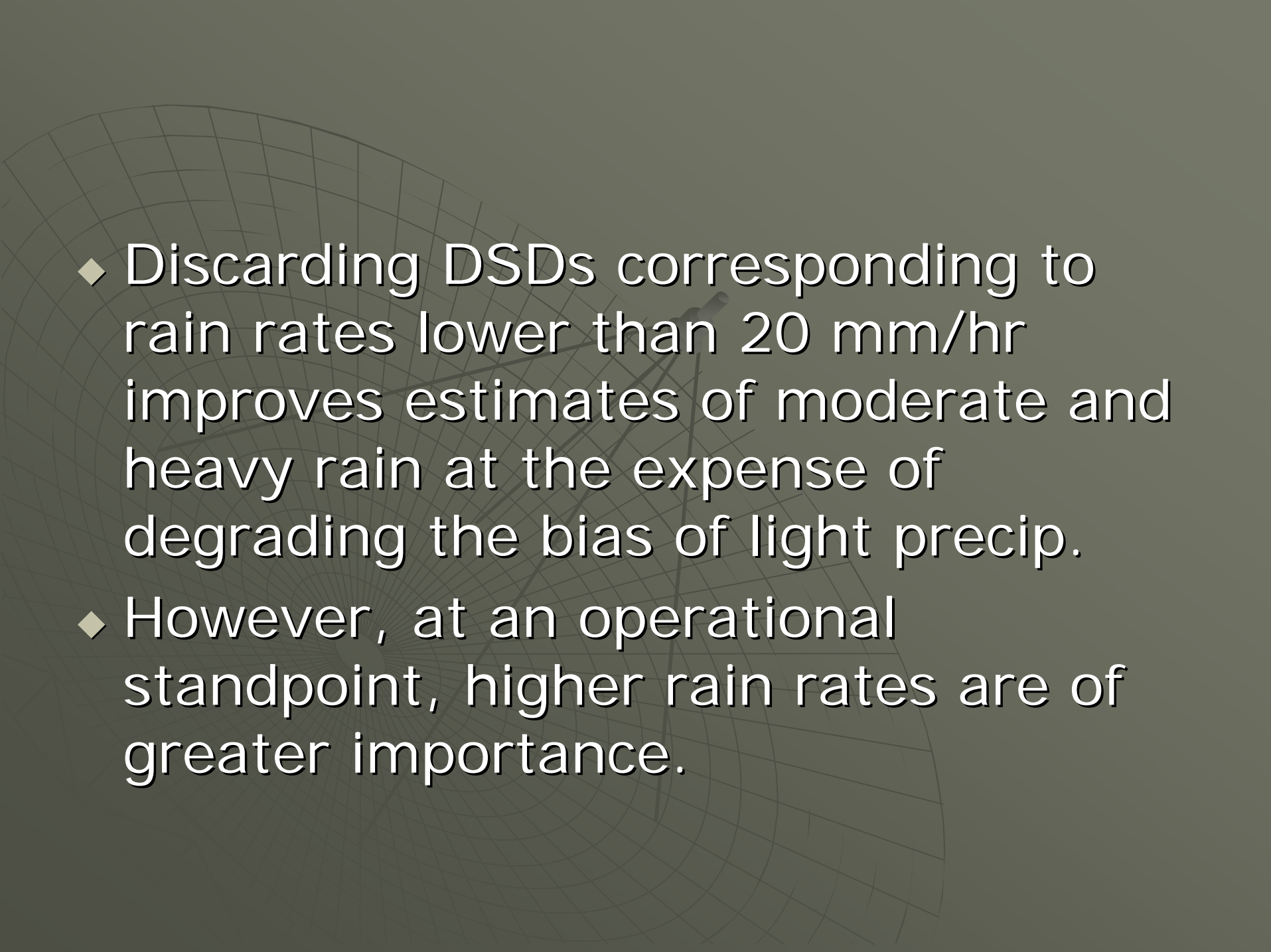


FIG. 1. Scatterplots of the rainfall-rate estimates  $R(K_{DP}, Z_{DR})$  versus the actual rainfall rate.

FIG. 2. Scatterplots of the rainwater content estimates  $M(K_{DP}, Z_{DR})$  versus the actual rainwater content.

Discarded rain rates lower than  
20 mm/hr

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- ◆ Discarding DSDs corresponding to rain rates lower than 20 mm/hr improves estimates of moderate and heavy rain at the expense of degrading the bias of light precip.
  - ◆ However, at an operational standpoint, higher rain rates are of greater importance.

# Comparison of different R and M estimators

- ◆  $R1 = 3.65 \times 10^{-2} Z_h^{0.625}$
- $R2 = 6.84 \times 10^{-3} Z_h^{-3.86} Z_v^{4.86}$
- $R3 = 40.56 K_{DP}^{0.866}$
- $R4 = 52.0 K_{DP}^{0.960} Z_{DR}^{-0.447}$

- ◆  $M1 = 3.44 \times 10^{-3} Z_h^{4/7}$
- $M2 = 1.06 \times 10^{-3} Z_h^{-3.32} Z_v^{4.18}$
- $M3 = 1.63 K_{DP}^{0.76}$
- $M4 = 3.11 K_{DP}^{0.918} Z_{DR}^{-0.764}$

TABLE 1. Standard deviations of rain rates for estimators  $\Delta R1$ ,  $\Delta R2$ ,  $\Delta R3$  given by Eqs. (9a)–(9c) and  $\Delta R4$  given by Eq. (7).

Standard deviation of the estimate (mm h <sup>-1</sup> )	Intervals of rain rate (mm h <sup>-1</sup> )			
	10–30	30–60	60–100	100–200
$\Delta R1$	8.5	15.3	24.5	34.0
$\Delta R2$	1.9	3.5	6.2	10.2
$\Delta R3$	3.5	6.4	10.9	16.3
$\Delta R4$	1.1	1.9	2.6	3.1

TABLE 2. Standard deviations of rainwater content for estimators  $\Delta M1$ ,  $\Delta M2$ ,  $\Delta M3$  given by Eqs. (10a)–(10c) and  $\Delta M4$  given by Eq. (8).

Standard deviation of the estimator (g m <sup>-3</sup> )	Intervals of rainwater content (g m <sup>-3</sup> )			
	1–2	2–3	3–4	4–5
$\Delta M1$	0.57	0.76	0.96	1.10
$\Delta M2$	0.27	0.38	0.43	0.48
$\Delta M3$	0.33	0.46	0.60	0.71
$\Delta M4$	0.16	0.23	0.25	0.22

R1 from  $Z=200R^{1.6}$   
 R2, R3 (Schidananda and Zrnica 1987)

M1 (Used in WSR-88D)  
 M2, M3 (Doviak and Zrnica 1993)

# Measurement errors impact on estimator performance

- ◆ Errors due to DSD and measurement errors.

Ryzhkov and Znic (1995)

TABLE 3. Standard deviations of rain-rate estimates caused by DSD ( $\Delta R_{DSD}$ ) variations, radar measurement error  $\Delta R_2$  for 2-km averaging in range, and  $\Delta R_4$  for 4-km averaging in range;  $\Delta R_{2,4}^{tot}$  are totals contributed by both DSD variations and measurement errors.

Estimator type	Rain rate (mm h <sup>-1</sup> )	$\Delta R_{DSD}$ (mm h <sup>-1</sup> )	$\Delta R_2$ (mm h <sup>-1</sup> )	$\Delta R_4$ (mm h <sup>-1</sup> )	$\Delta R_{2,4}^{tot}$ (mm h <sup>-1</sup> )	$\Delta R_4^{tot}$ (mm h <sup>-1</sup> )
$R(Z_{DR}, Z_v)$	30	2.7	13.5	9.3	13.8	9.7
	60	4.8	27.0	18.6	27.4	19.2
	100	8.2	45.0	31.0	45.7	32.1
$R(K_{DP})$	30	5.0	10.8	3.6	11.9	6.2
	60	8.7	14.5	4.8	16.9	9.9
	100	13.6	11.4	3.8	17.7	14.1
$R(K_{DP}, Z_{DR})$	30	1.5	12.0	4.2	12.1	4.5
	60	2.2	16.4	5.8	16.5	6.2
	100	2.8	13.5	5.3	13.8	6.0

Standard error of  $Z_{DR}$  is a function of:

- 1) Spectrum width
- 2) Correlation coefficient ( $\rho_{hv}(0)$ )
- 3) Number of sample pairs
- 4) Signal to noise ratio

$$\Delta R_{2,4}^{tot} = (\Delta R_{DSD}^2 + \Delta R_{2,4}^2)^{1/2}$$

$R(K_{DP}, Z_{DR})$  still exhibit the best performance provided  $K_{DP}, Z_{DR}$  are smoothed over a 4km range (16 gates).

# Comparison with rain gauges

- ◆ 9 June 1993, squall line passed over Washita River basin (42 rain gauges in a 603 km<sup>2</sup> area)
- ◆ Total rain accumulations at location of gauges computed from lowest scan of 0.4 deg

# Comparison with rain gauges

- ◆ Statistical noise in  $Z_h$  was reduced by averaging 4 successive range gates along radial ( $\sim 1\text{km}$ ). More needed for  $K_{DP}$  and  $Z_{DR}$ .
- ◆ Shifted radar rain field with respect to rain gauges.
- ◆ Hail was present; therefore reflectivity values larger than 53 dBZ were truncated to 53 dBZ (as in WSR-88D precip algorithm).

## Significant spread at high accumulations

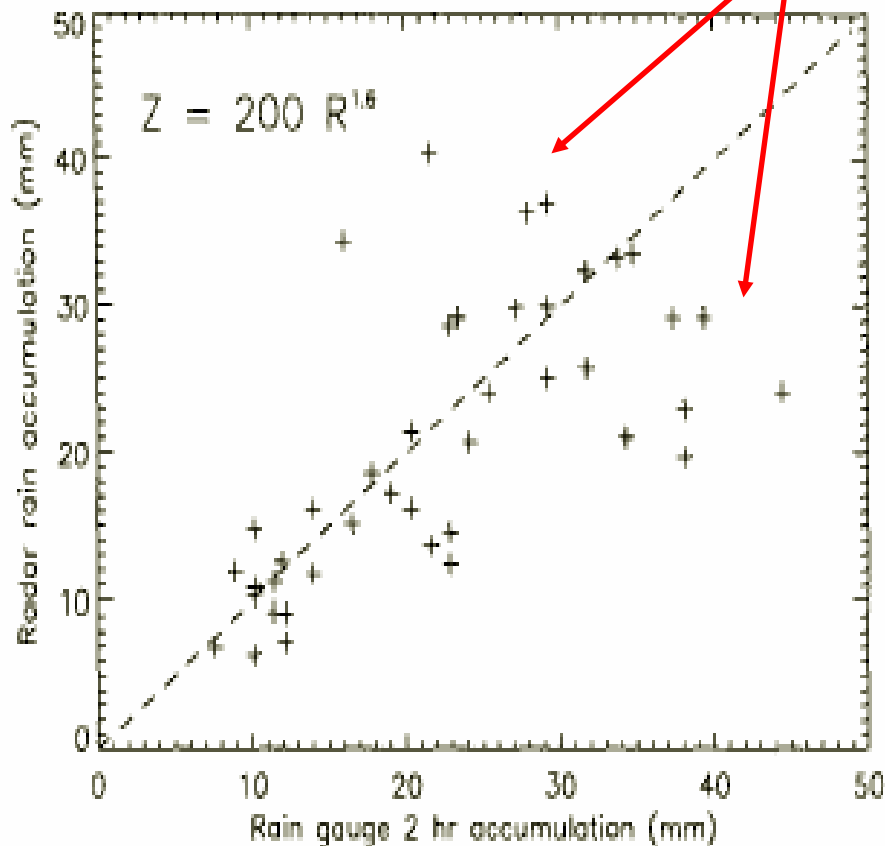


FIG. 3. Scattergram of cumulative rainfall obtained from  $R(Z_h)$  and the rain gauges.

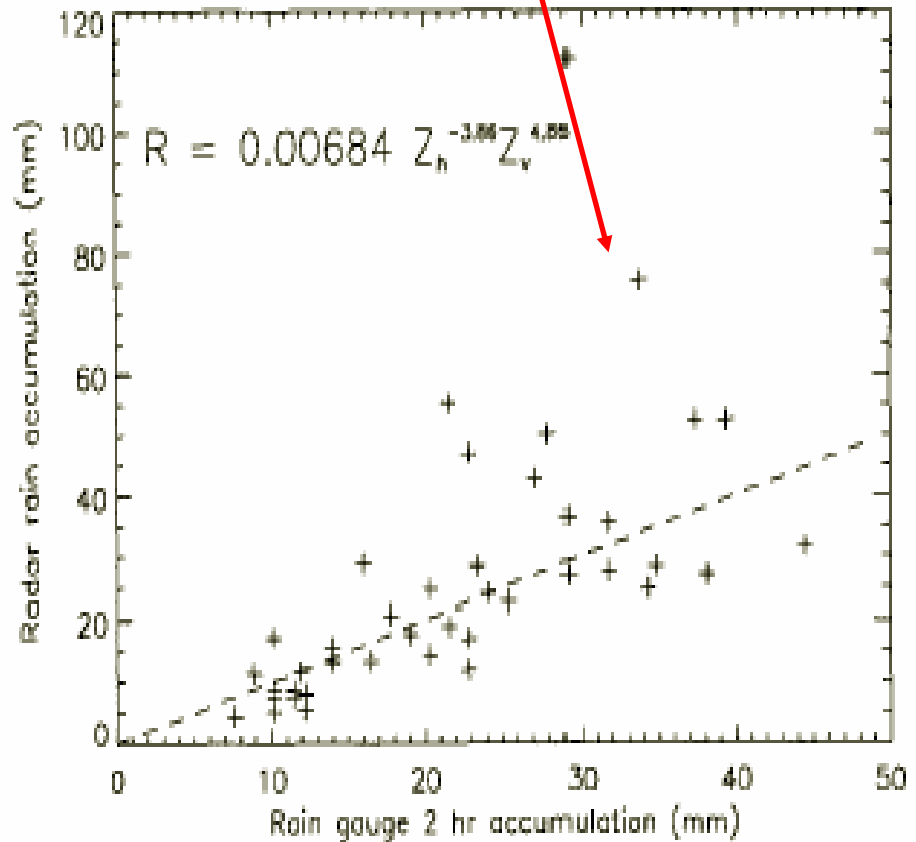


FIG. 4. Scattergram of cumulative rainfall obtained from  $R(Z_h, Z_v)$  and the rain gauges.

Reasons for bad performance:

- 1) Hail contamination (3 gauges removed)
- 2) Presence of melting hail aloft
  - ice cores  $\rightarrow$  drops larger than expected
  - Large  $Z_{dr}$   $\rightarrow$  underestimate  $R$

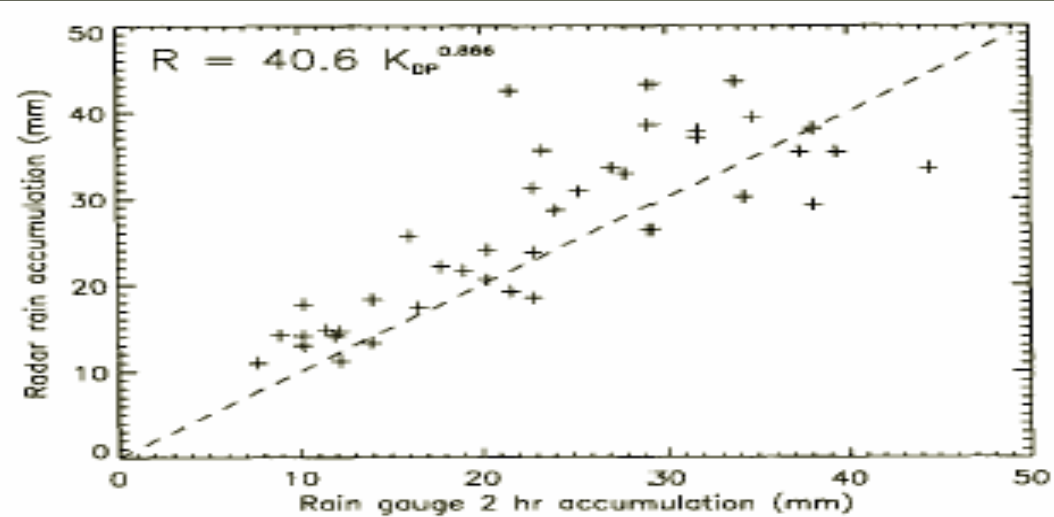


FIG. 5. Scattergram of cumulative rainfall obtained from  $R(K_{DP})$  and the rain gauges.

$K_{DP} > 0.4$  deg/km

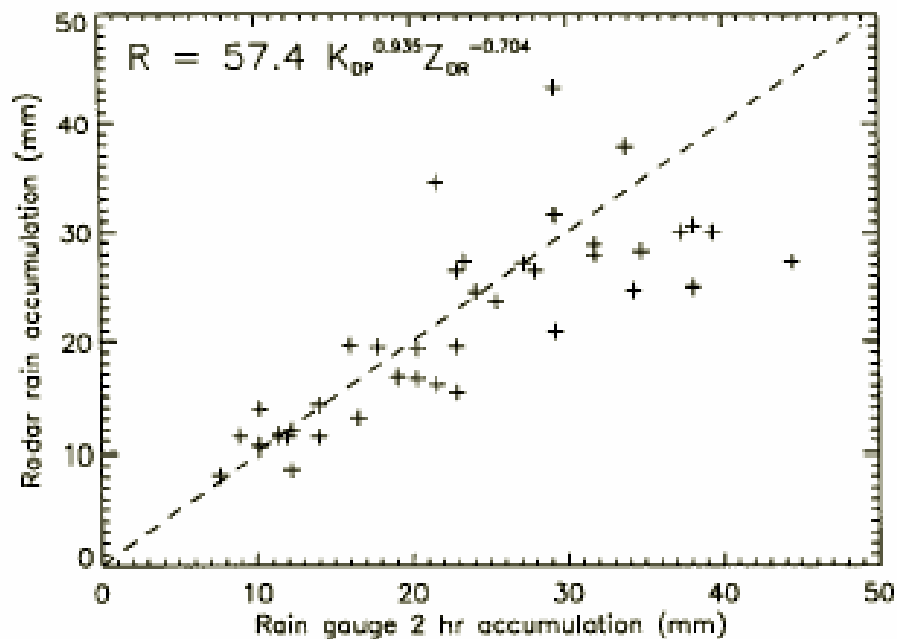


FIG. 6. Scattergram of cumulative rainfall obtained from  $R(K_{DP}, Z_{DR})$  [algorithm (5)] and the rain gauges.

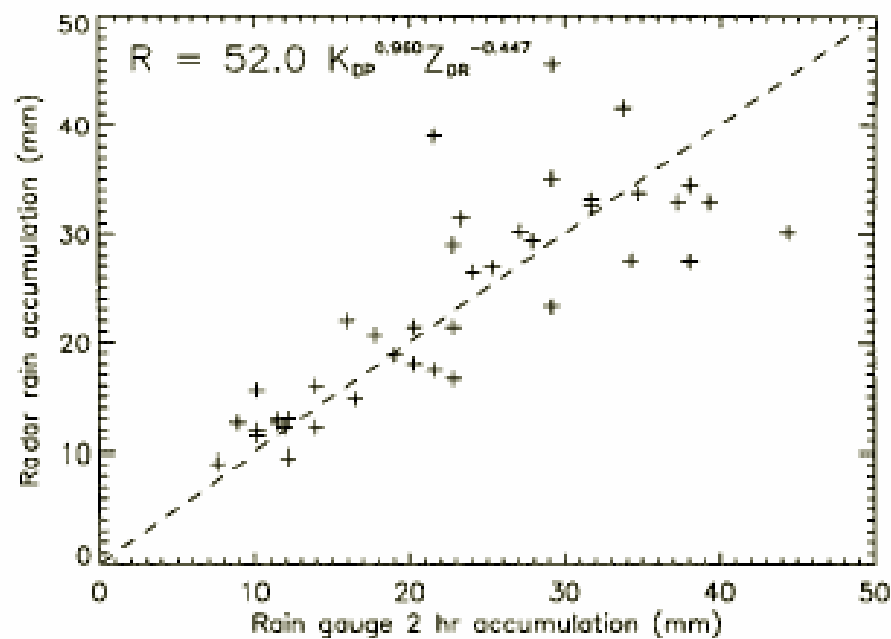


FIG. 7. Scatterplot of cumulative rainfall obtained from  $R(K_{DP}, Z_{DR})$  [algorithm (7)] and the rain gauges.

TABLE 4. The rms differences between radar rainfall estimators and rain gauges. All values in parentheses refer to estimates that exclude three locations where hail contamination was evident.

Algorithm	rms difference for 2-h total (mm)	rms percentage difference (%)
$Z_h = 200R^{1.6}$	8.0 (7.6)	33.3 (31.3)
$R = 6.84 \times 10^{-3} Z_h^{-3.86} Z_v^{4.86}$	17.4 (8.7)	65.1 (38.3)
$R = 40.6 K_{DP}^{-0.866}$	6.6 (5.3)	31.8 (27.6)
$R = 6.24 K_{DP}^{0.975} [1 - (Z_M/Z_v)^{-3/7}]^{-0.975}$	6.4 (4.7)	23.9 (18.2)
$R = 57.4 K_{DP}^{0.935} Z_{DR}^{-0.704}$	6.0 (4.5)	24.1 (18.7)
$R = 52.0 K_{DP}^{0.960} Z_{DR}^{-0.447}$	6.4 (4.5)	28.0 (22.1)

# Conclusions

- ◆ By varying parameters of a gamma DSD, calculations were made relating both rain rate and liquid water content to  $K_{DP}$  and  $Z_{DR}$
- ◆ Simulations show a relation  $R(K_{DP}, Z_{DR})$  produces a standard error of rain rate that is 2-3 times smaller than  $R(Z_h, Z_v)$  for **moderate and heavy** precipitation.

# Conclusions continued

- ◆ Based on these simulations R and M estimators that use  $K_{DP}$  and  $Z_{DR}$  are, in theory, virtually independent of DSD variations.
- ◆ In field comparison of these estimators with rain gauges reveals that the accuracy decreases due to measurement errors. Errors can be reduced with spatial smoothing of  $K_{DP}$  and  $Z_{DR}$ , however, bias errors in  $Z_{DR}$  still exist.
- ◆ All  $K_{DP}$  estimators out perform traditional rainfall measurements.