ATMO 689 Lecture #4 (09-23-04)

Backscattering matrix coefficients and radar observables for oblate spheroids

(DZ Ch. 8.5.2; BC Ch. 2)

• Backscattering matrix (S) revisited
  – Backscattering radar observables $Z_h$, $Z_v$, $Z_{dr}$, LDR as a function of $S$

• Rayleigh-Gans theory for scattering by oblate spheroids
  – Brief physical overview
  – Presentation of backscattering matrix elements for an oblate spheroid
Backscattering Matrix Equation Revisited

- The backscattered electric field ($E^b$) from a single hydrometeor for each polarization (h: horizontal and v: vertical) can be related to the incident electric field ($E^i$) for each polarization as a function of range ($r$) via the backscattering matrix ($S$) (note – $k_0$: wave number)
  - $S$ is an intrinsic (i.e., inherent) property of the hydrometeor causing the backscattering.
    - Size, shape, orientation, dielectric (i.e., phase, density)
  - Backscattered power and hence radar reflectivity for each polarization is proportional to the square of the appropriate $S$ term

\[
\begin{bmatrix}
    E_h \\
    E_v
\end{bmatrix}^b =
\begin{bmatrix}
    S_{hh} & S_{hv} \\
    S_{vh} & S_{vv}
\end{bmatrix}
\begin{bmatrix}
    E_h \\
    E_v
\end{bmatrix}^i e^{-j k_0 r}
\]

Horizontal reflectivity: $Z_h \sim |S_{hh}|^2$
Vertical reflectivity: $Z_v \sim |S_{vv}|^2$
Backscattering Radar Observables from Backscattering Matrix (S)

- Reflectivity (factor) at horizontal polarization ($z_h = z_{hh}$)
  - Co-polar $h$
  - $Z_h(\text{dBZ}) = 10 \log_{10}(z_h)$

- Reflectivity (factor) at vertical polarization ($z_v = z_{vv}$)
  - Co-polar $v$
  - $Z_v(\text{dBZ}) = 10 \log_{10}(z_v)$

- Differential reflectivity ($Z_{dr}$)
  - $(\text{Co-polar } h)/(\text{Co-polar } v)$
  - $Z_{dr} = 10 \log_{10}(z_h/z_v) = Z_h(\text{dBZ}) - Z_v(\text{dBZ})$

- Linear depolarization ratio ($LDR$)
  - $(\text{Cross-polar})/(\text{Co-polar})$
  - $LDR_{hv} = 10 \log_{10}(z_{hv}/z_{vv})$
  - $LDR_{vh} = 10 \log_{10}(z_{vh}/z_{hh})$
  - Theory of reciprocity: $LDR_{hv} = LDR_{vh} = LDR$
  - In practice: $LDR_{hv} \approx LDR_{vh} = LDR$

\[
Z_h = \left( \frac{4 \lambda^4}{\pi^4 |K_w|^2} \right) |S_{hh}|^2
\]
\[
Z_v = \left( \frac{4 \lambda^4}{\pi^4 |K_w|^2} \right) |S_{vv}|^2
\]
\[
Z_{dr} = 10 \log_{10} \left( \frac{|S_{hh}|^2}{|S_{vv}|^2} \right)
\]
\[
LDR_{hv} = 10 \log_{10} \left( \frac{|S_{hv}|^2}{|S_{vv}|^2} \right)
\]
\[
LDR_{vh} = 10 \log_{10} \left( \frac{|S_{vh}|^2}{|S_{hh}|^2} \right)
\]
Rayleigh-Gans Theory (Stephens Ch. 5; BC 1.3+2.3; DZ 8.5.2.4)

• Rayleigh scattering: radiation scattered by a single dipole, which you can consider a small spherical particle that is much smaller than the wavelength of the incident radiation.
  – Gans (1912) extended Lord Rayleigh’s theory to oblate and (and prolate) spheroids.
  – The small dipole oscillates at the frequency of the incident electromagnetic field, producing a secondary field that radiates out in all directions (i.e., the scattered field).
    • The incident electric field has two orthogonal components, \( E_{ih} \) (horizontal) and \( E_{iv} \) (vertical). Each component of incident electric field independently induces a dipole moment in the same polarization plane that creates that scattered electric field, or \( E^s_h \) and \( E^s_v \), respectively.
      – The resulting intensity of the excited dipole in each polarization plane is not only proportional to the incident electric field strength but also to the size, shape, orientation, and dielectric factor of the scattering particle.
    • For radar, we are often concerned with backscattering or radiation scattered 180° from the incident field (i.e., back to the radar).
      – \( E^b_h \) and \( E^b_v \), respectively.
Rayleigh-Gans Scattering Geometry

• Simplifications we will typically make
  – $\delta = 0^\circ$ (i.e., particle is aligned with polarization plane.)
  – $\theta = 0^\circ$ (zero radar elevation angle) for now
  – $\Psi$ is fixed (say $=0^\circ$)

• Other assumptions possible – probability distribution for orientation angles
  – to simulate LDR for hail (see BC 2.3.6 and BC Fig. 2.8c that follows)

KEY:

$v$: vertical
$h$: horizontal
$\delta$: angle between the propagation direction of incident field and symmetry axis ($n$)
$\Psi$: canting angle or angle between incident electric field and the projection of the axis of symmetry on the polarization plane
$\theta$: radar elevation angle;
Rayleigh Gans Theory: Backscattering Matrix Elements

• The backscattering matrix elements for an oblate spheroid (e.g., raindrop) are given by (p. 249 DZ)

\[ S_{hh} = k_0^2 [(p_v - p_h) \sin^2(\delta) \sin^2(\Psi) + p_h] \]
\[ S_{vv} = k_0^2 [(p_v - p_h) \cos^2(\delta) \cos^2(\Psi) + p_h] \]
\[ S_{hv} = 0.5k_0^2(p_v - p_h) \sin^2(\delta) \sin(2\Psi) \]

where

\[ e: \text{eccentricity} \]
\[ a/b: \text{(minor/major) axis ratio} \]
\[ m: \text{refractive index (more later)} \]

\[ p_{h,v} = \frac{ab^2}{3} \left[ \frac{(m^2 - 1)}{A_{h,v} (m^2 - 1) + 1} \right] \]
\[ A_v = \frac{1}{e^2} \left[ 1 - \left( \frac{1 - e^2}{e^2} \right)^{1/2} \sin^{-1}(e) \right] = 1 - 2A_h \]

\[ e = \left[ 1 - (a/b)^2 \right]^{1/2} \]

If \( \delta = \Psi = 0^\circ \),
You too can calculate $Z_{dr}$ and LDR!

**Note**: isotropic or random (not fixed) orientation for LDR orientation for LDR

For Oblate Spheroids from Gans Theory.

- *next time* – refractive index of water, ice, and ice/air mixtures

Bringi and Chandrasekar (2001)
Differential Reflectivity – Ice (dielectric effects)

- Response of $Z_{dr}$ to hydrometeor shape for ice is very different than for water drops.
  - Shapes of ice particles and water drops are different
  - Also, $Z_{dr}$ sensitivity to hydrometeor shape varies with dielectric constant of the scattering hydrometeors.
    - Since dielectric constant of ice is about 20% that of water, particle shape has a much smaller effect on $Z_{dr}$ measurement in ice than in liquid water hydrometeors.
    - Inclusion of air in ice particles of low bulk density (e.g., snow) lowers effective dielectric constant further.

- Fig. 2: $Z_{dr}$ (dB) vs. axis ratio (a/b) for oblate spheroids of varying dielectric.
  - Rain
  - Solid ice (hail, ice crystals; 0.9 g cm$^{-3}$)
  - Graupel (bulk density: 0.3-0.6 g cm$^{-3}$)
  - Snow (bulk density: 0.03-0.12 g cm$^{-3}$)

- Canting can also decrease $Z_{dr}$
  - Mean canting angle in rain is zero
  - Hail wobbles and spins in descent