ATMO 689 Lecture #4 (09-23-04)

Backscattering matrix coefficients and radar observables for oblate spheroids

(DZ Ch. 8.5.2; BC Ch. 2)

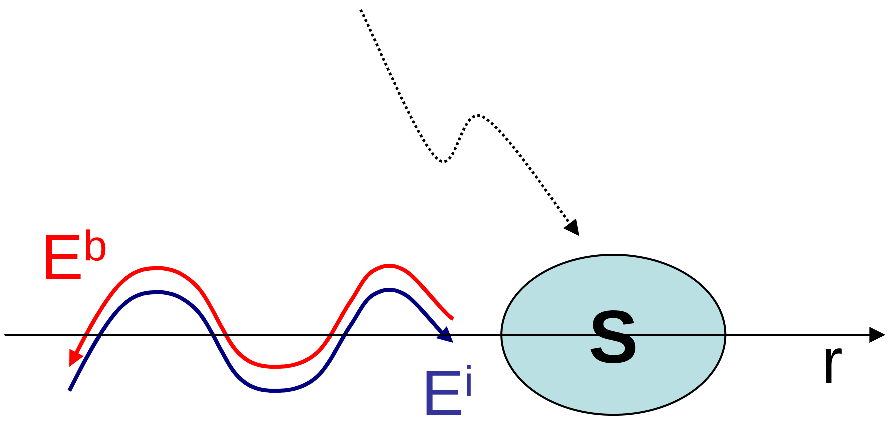
- Backscattering matrix (S) revisited
  - Backscattering radar observables Z_h, Z_v, Z_{dr} ,LDR as a function of S
- Rayleigh-Gans theory for scattering by oblate spheroids
  - Brief physical overview
  - Presentation of backscattering matrix elements for an oblate spheroid

Backscattering Matrix Equation Revisited

- The backscattered electric field (E^b) from a single hydrometeor for each polarization (h: horizontal and v: vertical) can be related to the incident electric field (E^i) for each polarization as a function of range (r) via the backscattering matrix (S) (note – k_0: wave number)
  - S is an intrinsic (i.e., inherent) property of the hydrometeor causing the backscattering.
    - Size, shape, orientation, dielectric (i.e., phase, density)
  - Backscattered power and hence radar reflectivity for each polarization is proportional to the square of the appropriate S term

Backscatter Matrix (S)

$$\begin{bmatrix} E_h \\ E_v \end{bmatrix}^b = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} E_h \\ E_v \end{bmatrix}^i \frac{e^{-jk_0 r}}{r}$$



Horizontal reflectivity: $Z_h \sim |S_{hh}|^2$
Vertical reflectivity: $Z_v \sim |S_{vv}|^2$

Backscattering Radar Observables from Backscattering Matrix (S)

- Reflectivity (factor) at horizontal polarization ($z_h \equiv z_{hh}$)
  - Co-polar h
  - Z_h(dBZ)=10LOG$_{10}$(z_h)

$$z_h = \left(\left(4\lambda^4\right) / \left(\pi^4 \left|K_w\right|^2\right)\right) \left|S_{hh}\right|^2$$

- Reflectivity (factor) at vertical polarization ($z_v \equiv z_{vv}$)
  - Co-polar v
  - Z_v(dBZ)=10LOG$_{10}$(z_v)

$$z_v = \left(\left(4\lambda^4\right) / \left(\pi^4 \left|K_w\right|^2\right)\right) \left|S_{vv}\right|^2$$

- Differential reflectivity (Zdr)
  - (Co-polar h)/(Co-polar v)
  - Z_{dr}= 10LOG$_{10}$(z_h/z_v)] = Z_h(dBZ) – Z_v(dBZ)

$$Z_{dr} = 10LOG_{10}\left(\frac{\left|S_{hh}\right|^2}{\left|S_{vv}\right|^2}\right)$$

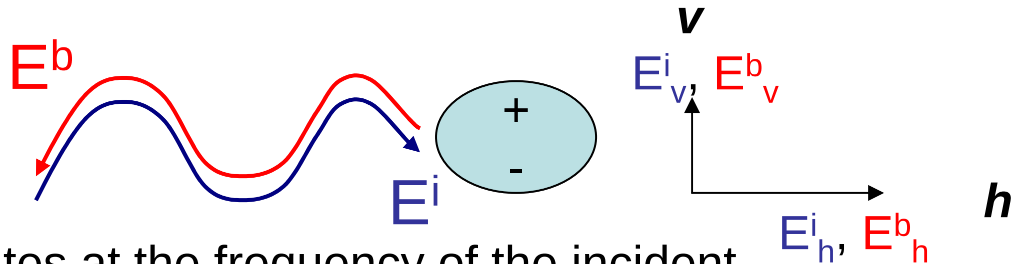
- Linear depolarization ratio (LDR)
  - (Cross-polar)/(Co-polar)
  - LDR_{hv} = 10LOG$_{10}$(z_{hv}/z_{vv})
  - LDR_{vh} = 10LOG$_{10}$(z_{vh}/z_{hh})
  - Theory of reciprocity: LDR_{hv} = LDR_{vh} = LDR
  - In practice: $LDR_{hv} \approx LDR_{vh}$ = LDR

$$LDR_{hv} = 10LOG_{10}\left(\frac{\left|S_{hv}\right|^2}{\left|S_{vv}\right|^2}\right)$$

$$LDR_{vh} = 10LOG_{10}\left(\frac{\left|S_{vh}\right|^2}{\left|S_{hh}\right|^2}\right)$$

Rayleigh-Gans Theory (Stephens Ch. 5; BC 1.3+2.3; DZ 8.5.2.4)

- Rayleigh scattering: radiation scattered by a single dipole, which you can consider a small *spherical* particle that is much smaller than the wavelength of the incident radiation.
  - Gans (1912) extended Lord Rayleigh's theory to *oblate* and (and *prolate) spheroids*.



  - The small dipole oscillates at the frequency of the incident electromagnetic field, producing a secondary field that radiates out in all directions (i.e., the scattered field).
    - The incident electric field has two orthogonal components, E^i_h (horizontal) and E^i_v (vertical). Each component of incident electric field independently induces a dipole moment in the same polarization plane that creates that scattered electric field, or E^s_h and E^s_v, respectively .
      - The resulting intensity of the excited dipole in each polarization plane is not only proportional to the incident electric field strength but also to the size, shape, orientation, and dielectric factor of the scattering particle
    - For radar, we are often concerned with backscattering or radiation scattered 180° from the incident field (i.e., back to the radar).
      - E^b_h and E^b_v, respectively

Rayleigh-Gans Scattering Geometry

- Simplifications we will *typically* make
  - $\delta = 0°$ (i.e., particle is aligned with polarization plane.)
  - $\theta = 0°$ (zero radar elevation angle) for now
  - Ψ is fixed (say $=0°$)
- Other assumptions possible – probability distribution for orientation angles
  - to simulate LDR for hail (see BC 2.3.6 and BC Fig. 2.8c that follows)

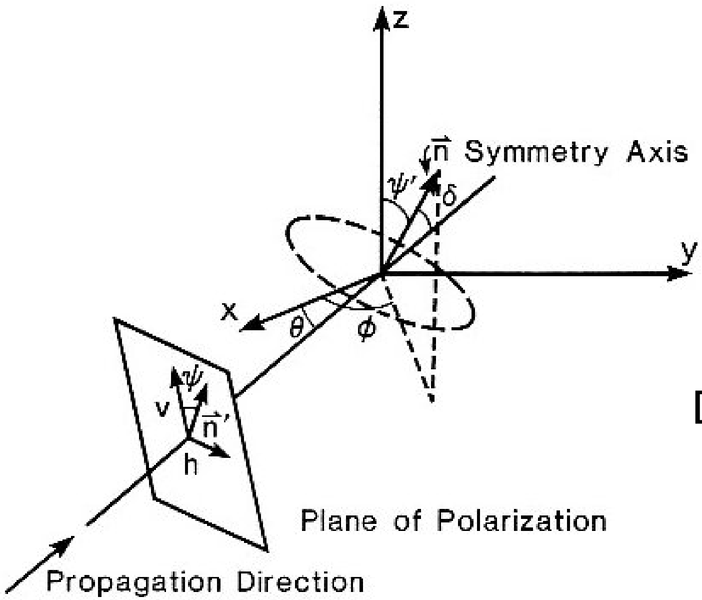


Fig. 8.15 Scattering geometry where **n** is the symmetry axis of the scatter, **n'** is the projection of **n** onto the constant phase plane, θ is the radar elevation angle, **h** and **v** are the linear polarization base vectors, and ψ is the canting angle of the scatterer. The vector **v** denoting vertical polarization is in the x, z plane.

Doviak and Zrnic (1993)

KEY:

v: vertical

h: horizontal

δ: angle between the propagation direction of incident field and symmetry axis (n)

Ψ: canting angle or angle between incident electric field and the projection of the axis of symmetry on the polarization plane

θ: radar elevation angle;

Rayleigh Gans Theory: Backscattering Matrix Elements

- The backscattering matrix elements for an oblate spheroid (e.g., raindrop) are given by (p. 249 DZ)

$$p_{h,v} = \frac{ab^2}{3}\left[\frac{(m^2-1)}{A_{h,v}(m^2-1)+1}\right]$$

$$A_v = \frac{1}{e^2}\left[1-\left(\frac{1-e^2}{e^2}\right)^{1/2}\sin^{-1}(e)\right] = 1-2A_h$$

$$e = \left[1-(a/b)^2\right]^{1/2}$$

- $S_{hh} = k_0^2[(p_v-p_h)\sin^2(\delta)\sin^2(\Psi) + p_h]$
- $S_{vv} = k_0^2[(p_v-p_h)\cos^2(\delta)\cos^2(\Psi) + p_h]$
- $S_{hv} = 0.5k_0^2(p_v-p_h)\sin^2(\delta)\sin(2\Psi)$

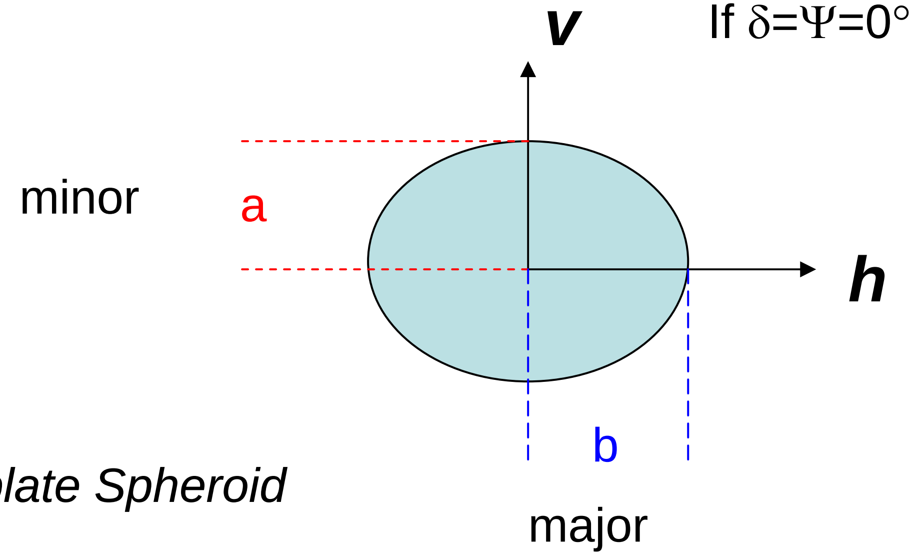
where

e: eccentricity

a/b: (minor/major) axis ratio

m: refractive index (more later)

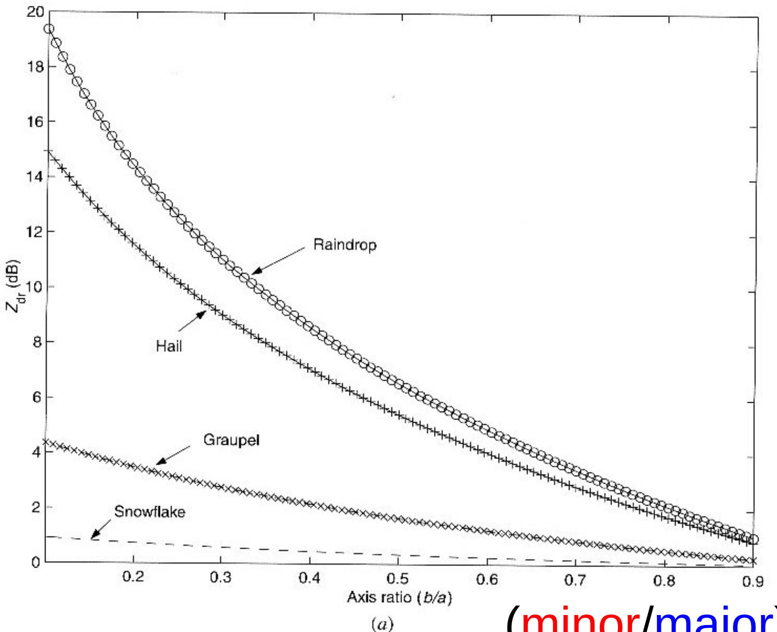
If $\delta=\Psi=0°$,



Oblate Spheroid

You too can calculate Z_{dr} and LDR!

Zdr (dB)



(minor/major)

Fig. 2.8. (a) Differential reflectivity (Z_{dr}) of a single particle of oblate shape versus axis ratio. Several particle types with different densities are illustrated to show the strong weighting of Z_{dr} on the dielectric constant for a given (b/a). (b) Summary of typical Z_{dr} values of raindrops of various sizes and hail. The black arrows on the hail particle represent the tumbling motions as it falls in a thunderstorm. Adapted from Wakimoto and Bringi (1988). (c) Linear depolarization ratio of a single oblate particle with isotropic orientation distribution, see (2.72), versus axis ratio. Particle types are as in Fig. 2.8a. Note strong weighting of LDR with dielectric constant (or density) for a fixed axis ratio.

LDR (dB)

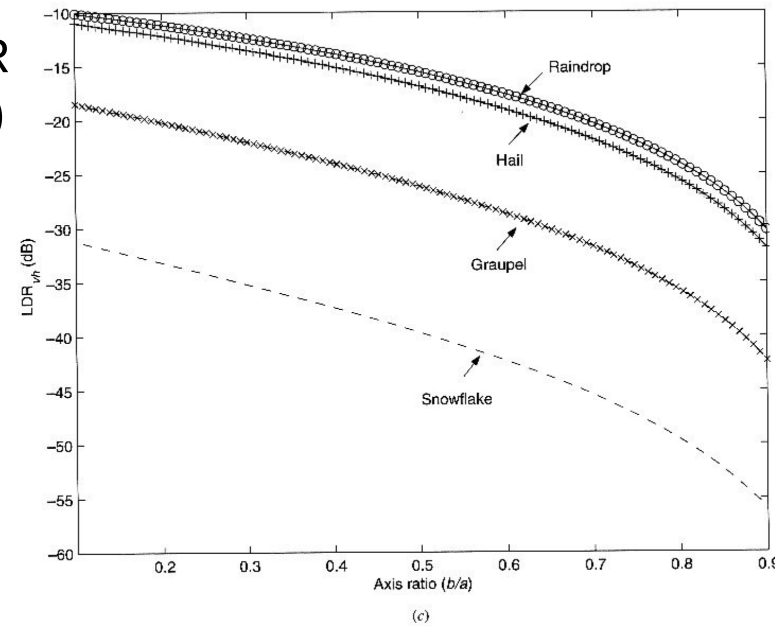


Fig. 2.8. (cont.)

(minor/major)

Note: isotropic or random (not fixed) orientation for LDR

For Oblate Spheroids from Gans Theory.

- *next time* – refractive index of water, ice, and ice/air mixtures

Bringi and Chandrasekar (2001)

Differential Reflectivity – Ice (dielectric effects)

- Response of Z_{dr} to hydrometeor shape for ice is very different than for water drops.
  - Shapes of ice particles and water drops are different
  - Also, Z_{dr} sensitivity to hydrometeor shape varies with dielectric constant of the scattering hydrometeors.
    - Since dielectric constant of ice is about 20% that of water, particle shape has a much smaller effect on Z_{dr} measurement in ice than in liquid water hydrometeors.
    - Inclusion of air in ice particles of low bulk density (e.g., snow) lowers effective dielectric constant further.
- Fig. 2: Z_{dr} (dB) vs. axis ratio (a/b) for oblate spheroids of varying dielectric.
  - Rain
  - Solid ice (hail, ice crystals; 0.9 g cm^{-3})
  - Graupel (bulk density: 0.3-0.6 g cm^{-3})
  - Snow (bulk density: 0.03-0.12 g cm^{-3})
- Canting can also decrease Z_{dr}
  - Mean canting angle in rain is zero
  - Hail wobbles and spins in descent

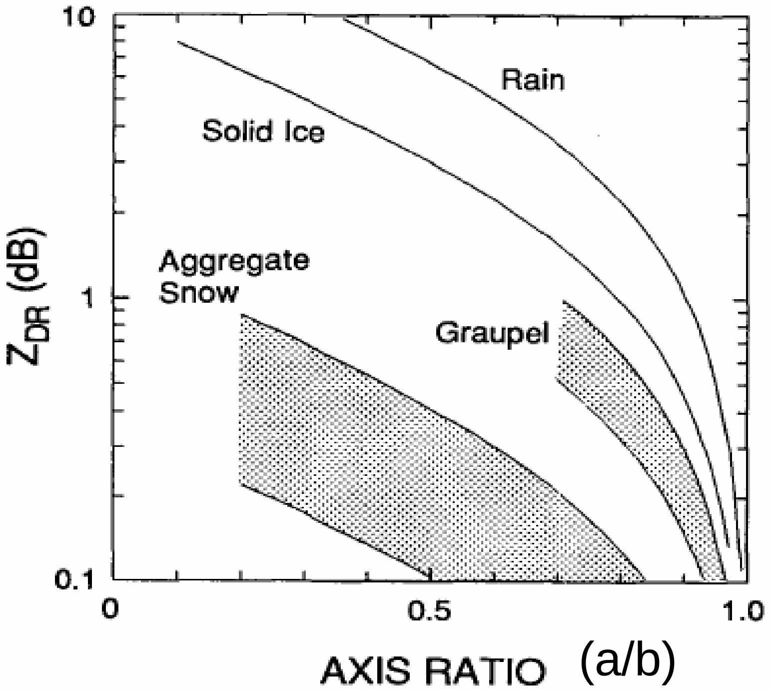


FIG. 2. Calculations of Z_{DR} in decibels as a function of particle axis ratio for spheroids having the effective dielectric properties of raindrops, solid ice, graupel, and aggregate snow. As axis ratio decreases from 1 toward 0, particle (spheroid) shape becomes more oblate. Curves shown for graupel cover bulk densities of 0.3 g cm^{-3} (lower boundary of shaded region) to 0.6 g cm^{-3} (upper boundary). Values shown for aggregate snow cover bulk densities of 0.03 g cm^{-3} (lower boundary of shaded region) to 0.12 g cm^{-3} (upper boundary). Calculations use the scattering theory of Gans (1912).