ATMO 689 Lecture #3 (09-21-04)

Radar Waves, Polarization, and Scattering*

- Electromagnetic Spectrum
- Electromagnetic Waves – Brief Mathematical Description
- Polarization
- Backscattering Matrix
- Covariance Matrix
- Radar observables

* Several slides courtesy of W. A. Petersen of UAH

Electromagnetic Spectrum

$c = f \bullet \lambda$

c: speed of light

$c = 3 \times 10^8$ m s^{-1}

f: frequency

λ: wavelength

For **radar meteorology**, we are concerned with the microwave frequencies (roughly 1-100 GHz)

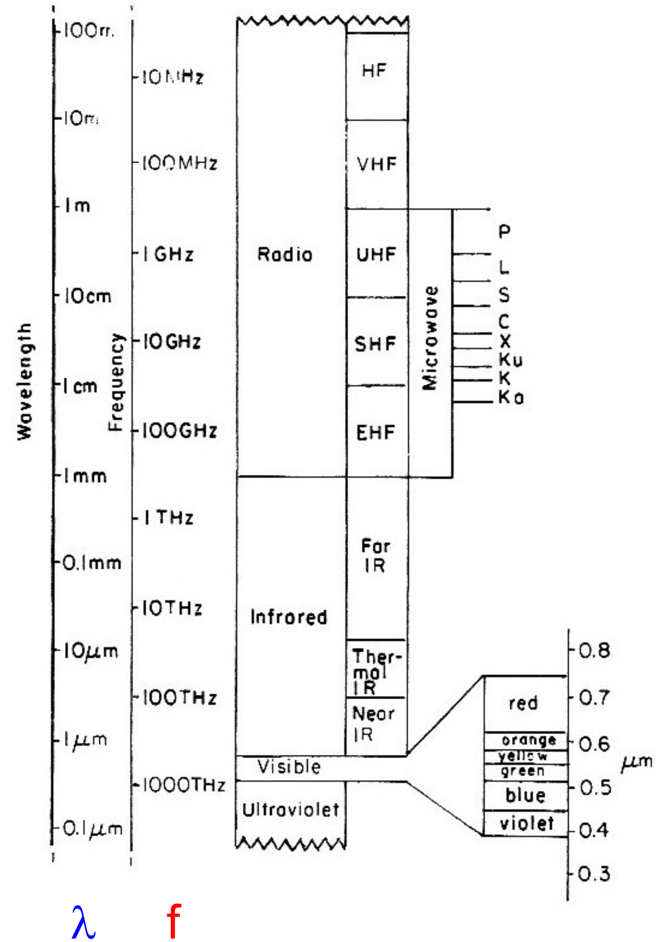
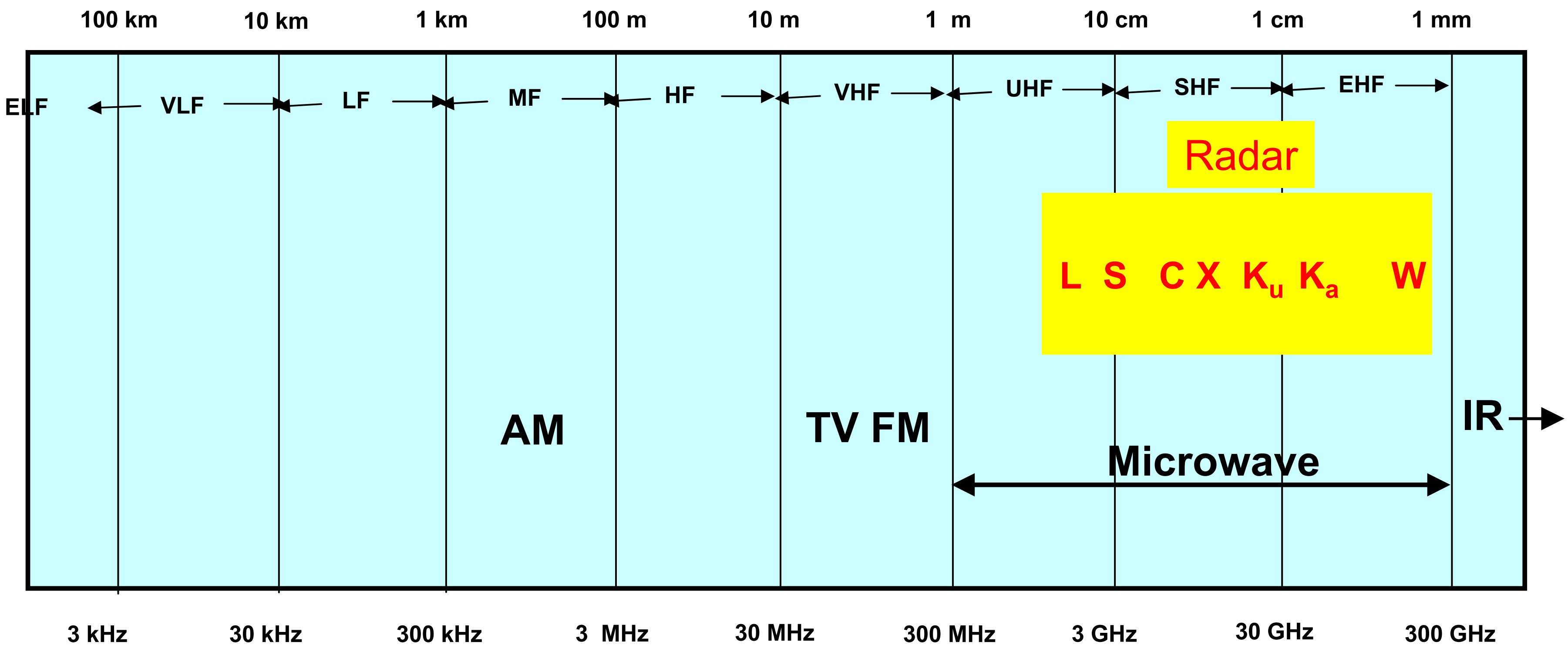


Figure 2.3 The electromagnetic spectrum. The diagram shows those parts of the electromagnetic spectrum which are important in remote sensing, together with the conventional names of the various regions of the spectrum. The letters (P, L, S, etc.) used to denote parts of the microwave spectrum are in common use in remote sensing, being standard nomenclature among radar engineers in the United States. Various terminologies are in use for the subdivisions of the infrared (IR) part of the spectrum. That adopted here defines the thermal band as lying between 3 and 15 μm, since this region contains most of the power emitted by black bodies at terrestrial temperatures.

Stephens (1994)

Electromagnetic Waves: Spectrum



We are concerned with the microwave portion of the spectrum in Radar Meteorology

L-band (e.g., 30 cm):	NOAA wind profilers; some ATC radars	Precipitation
S-band (10 cm):	S-pol, CSU-CHILL, N-pol, WSR-88D, WSR-57, ASR-9 (ATC)	
C-band (5 cm):	C-pol, DLR, MIT, TOGA, SMART-R, WSR-74C	
X-band (3 cm):	X-pol, DOW, EDOP, ELDORA, NOAA-P3 tail, aircraft wx avoidance, (many others)	
Ku-band (2 cm):	TRMM PR, GPM-PRII (dual-freq.)	
Ka-band (.86 cm):	NOAA-ETL; GPM-PRII (dual-freq.)	Cloud
W-band (3.2 mm):	NASA-JPL, UMASS, U. Wyoming	

Electromagnetic Waves (Stephens Ch. 2, DZ Ch. 2)

- Electromagnetic waves are generated by oscillating (i.e., time varying) electric charges, which, in turn, generate an oscillating electric field.

- Through Maxwell's Equations, we know that an oscillating Electric Field produces an accompanying oscillating magnetic field and hence proceeds to propagate outward from the original charge.

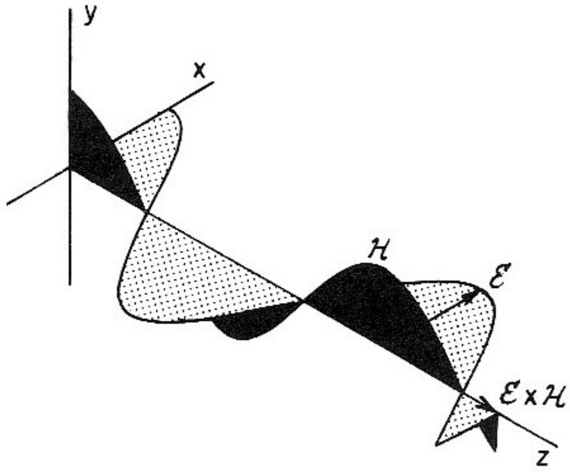
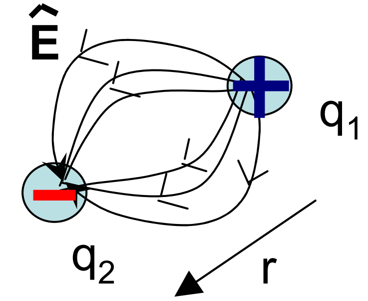


Figure 2.1 A schematic view of a time harmonic electromagnetic wave propagating along the z axis. The oscillating electric $\mathcal{E}$ and magnetic $\mathcal{H}$ fields are shown. Note that the oscillations are in the x–y plane and perpendicular to the direction of propagation.

Stephens (1994)

Basic Review of Electromagnetic Waves (DZ Ch. 2)

$\hat{\mathbf{E}} = \hat{\mathbf{F}}/q \quad \hat{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 \hat{\mathbf{r}}}{r^2}$



Electric Field Static Case

Electric Field (E) = Electric Force (F) / unit charge (q)

Field lines point from positive to negative charge

Flux of field through a unit area = $\hat{\mathbf{\Phi}}_E$

Magnetic Field: $\hat{\mathbf{B}}$

Biot-Savart Law $\hat{\mathbf{B}} = \frac{\mu_0 I}{4\pi}\frac{(d\hat{\mathbf{s}} \times \hat{\mathbf{r}})}{r^2}$

Requires electric current- moving charge and is $\perp \hat{\mathbf{E}}$

Ampere's Law $\oint \hat{\mathbf{B}} \cdot d\hat{\mathbf{s}} = m_0 I + \mu_0\varepsilon_0 d\hat{\mathbf{\Phi}}_E/dt$

Or changing electric flux (e.g., an oscillating electric field)

Oscillations in voltage-current produce oscillating **B** and **E**. Part of energy is radiated as propagating, oscillating electromagnetic wave in the **direction of the Poynting Vector: S** = $1/\mu_0$ **E** X **B** (W/m^2: rate of energy flow through a surface)

Faraday's Law for induction via magnetic flux $\hat{\mathbf{\Phi}}_m$

$\oint \hat{\mathbf{E}} \cdot d\hat{\mathbf{s}} = -d\hat{\mathbf{\Phi}}_m/dt$

Faraday, Ampere's Laws two of Maxwell's Eq. governing EM wave propagation

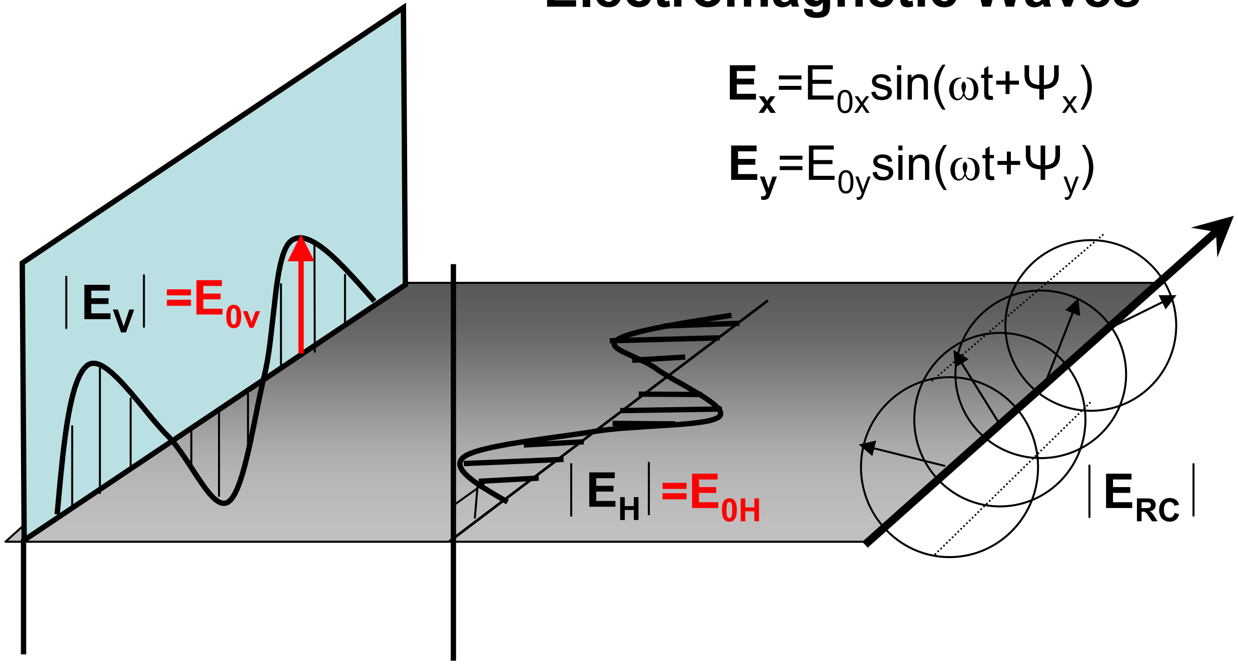
Changing **B** and **E** fields force EM wave propagation- these are radiated by/through wave guide and antenna feedhorn then focused by the radar antenna

EM waves move at speed of light: **c= 2.998 x 10^8** m/s ; recall **c=λf**

Electromagnetic Waves

$$\mathbf{E_x} = E_{0x}\sin(\omega t + \Psi_x)$$

$$\mathbf{E_y} = E_{0y}\sin(\omega t + \Psi_y)$$



E_{0i} = amplitude of E-field in plane *i* ; Ψ_i = phase of wave in plane *i*

HORIZONTAL POLARIZATION

When $\Psi_x - \Psi_y = n\pi$, n=0,1,2,3,….. then wave is Linearly polarized (if $E_{0y}=0$, horizontally polarized, $\mathbf{E_H}$, if $E_{0x}=0$, vertically polarized, $\mathbf{E_V}$)

CIRCULAR AND ELLIPTICAL POLARIZATION

When $E_{0x} = E_{0y}$, but $\Psi_x \neq \Psi_y$ and $\Psi_x - \Psi_y = \pi/2$, right hand circularly polarized $\mathbf{E_{RC}}$ (E-field rotates about propagation direction in a right handed fashion), if delta-phase angle = $-\pi/2$, left hand circular polarized, otherwise elliptically polarized.

Scattering and the Backscattering Matrix (DZ Ch. 8; BC Ch. 3)

- Define time-varying electric field (E^i) that is incident (i) on a particle

$$\vec{E}^i(t) = \mathrm{Re}\left(\vec{E}^i e^{j\omega t}\right) = \hat{h}_i \left|E_h^i\right| \cos\left(\omega t + \theta_h^i\right) + \hat{v}_i \left|E_v^i\right| \cos\left(\omega t + \theta_v^i\right)$$

where h,v: horizontal and vertical polarization; $\omega=2\pi f$ (angular Doppler shift frequency); and θ: offset phase

- The scattered electric field (E^s) by a single particle is

$$E^s = S\,E^i$$

where S is the backscattering matrix.

• Since $\vec{E}^s = E_h^s \hat{h}_s + E_v^s \hat{v}_s$

we can also express the scattered electric field for each polarization component, since

$$E_h^s(t) = \left[\sum_m |S_{hh}^m| e^{j\delta_{hh}^m} e^{-j(\omega_m t + \theta_m)} \right] e^{j\omega t} = \left[\gamma_{hh}(t) e^{j\alpha_{hh}(t)} \right] e^{j\omega t}$$

$$E_v^s(t) = \left[\sum_m |S_{vh}^m| e^{j\delta_{vh}^m} e^{-j(\omega_m t + \theta_m)} \right] e^{j\omega t} = \left[\gamma_{vh}(t) e^{j\alpha_{vh}(t)} \right] e^{j\omega t}$$

where we have assumed that

$$E_h^i = 1 \quad and \quad E_v^i = 0$$

(i.e., we assumed incident wave had horizontal polarization) and we have summed over all particles (m) in the radar resolution volume.

• Dropping jωt term for convenience for now, we get

$$E_h^s(t) = \sum_m E_h^{s(m)} = \gamma_{hh}(t) e^{j\alpha_{hh}(t)}$$

$$E_v^s(t) = \sum_m E_v^{s(m)} = \gamma_{vh}(t) e^{j\alpha_{vh}(t)}$$

• Keep in mind that we could have gone through the same exercise for incident vertical polarization. Combining it all together and using the fact that hv = vh for reciprocal media like precipitation, we can define the backscatter (BSA) matrix as

$$\mathbf{S}_{\mathrm{BSA}}(t) = \begin{bmatrix} \gamma_{hh}(t) e^{j\alpha_{hh}(t)} & \gamma_{hv}(t) e^{j\alpha_{hv}(t)} \\ \gamma_{hv}(t) e^{j\alpha_{hv}(t)} & \gamma_{vv}(t) e^{j\alpha_{vv}(t)} \end{bmatrix}_{\mathrm{BSA}}$$

• If we put it all together, we have a relationship between the scattered electric field and the incident electric field in terms of the backscattering matrix S_{BSA}

$$\begin{bmatrix} E_h \\ E_v \end{bmatrix}^b = S_{BSA}(t) \begin{bmatrix} E_h \\ E_v \end{bmatrix}^i e^{j\omega t}$$

• Radar does not measure the electric field. Instead, signal voltages V are processed in response to the backscattered electric field being intercepted by the antenna. However, voltage is proportional to the RHS of the above equation so

$$\begin{bmatrix} V_h(t) \\ V_v(t) \end{bmatrix} \sim S_{BSA}(t) \begin{bmatrix} E_h \\ E_v \end{bmatrix}^i$$

• The mean voltage is zero as you integrate over time. So, radar meteorologists use various *second-order moments* of the voltage $(V_{ij}V_{kl}^*)$ instead. These voltage moments are related to scattering coefficients of the particles. Without deriving here, we merely provide those scattering coefficients in a 3x3 matrix called the **covariance matrix.** The covariance matrix is a complete polarimetric characterization of the particles in the RRV.

What are we measuring? The time-averaged (indicated by overbar), backscatter **POLARIZATION COVARIANCE MATRIX**. (BC pg. 134-135)

$$= \begin{bmatrix} \langle \gamma_{hh}^2 \rangle & \sqrt{2}\langle \gamma_{hh}\gamma_{hv} e^{j(\alpha_{hh}-\alpha_{hv})} \rangle & \langle \gamma_{hh}\gamma_{vv} e^{j(\alpha_{hh}-\alpha_{vv})} \rangle \\ \sqrt{2}\langle \gamma_{hv}\gamma_{hh} e^{j(\alpha_{hv}-\alpha_{hh})} \rangle & 2\langle \gamma_{hv}^2 \rangle & \sqrt{2}\langle \gamma_{hv}\gamma_{vv} e^{j(\alpha_{hv}-\alpha_{vv})} \rangle \\ \langle \gamma_{vv}\gamma_{hh} e^{j(\alpha_{vv}-\alpha_{hh})} \rangle & \sqrt{2}\langle \gamma_{vv}\gamma_{hv} e^{j(\alpha_{vv}-\alpha_{hv})} \rangle & \langle \gamma_{vv}^2 \rangle \end{bmatrix}$$

Note- the matrix is symmetric; there are only six unique elements measured

$\gamma_{\mathbf{hh,vv}}$ = instantaneous scattering (proportional to received voltage amplitude) = f(η); complex terms

hh= receive/ transmit horizontal

vv= receive/transmit vertical

hv=receive horizontal, transmit vertical (reciprocity says hv = vh; just the cross-polar return)

vh=receive vertical, transmit horizontal

α**(t)** = phase information

Also commonly written as an

Ensemble average of complex back scattering terms

Treat $S_{xx,\,xy}$ as proportional to the returned/measured signal voltage with amplitude and phase information in each polarization.

$$\begin{pmatrix} |S_{hh}|^2 & \sqrt{2}(S_{hh}S_{hv}^*) & S_{hh}S_{vv}^* \\ \sqrt{2}(S_{hv}S_{hh}^*) & 2|S_{hv}|^2 & \sqrt{2}(S_{hv}S_{vv}^*) \\ S_{vv}S_{hh}^* & \sqrt{2}(S_{vv}S_{hv}^*) & |S_{vv}|^2 \end{pmatrix}$$

And the terms of the covariance matrix yield:

$$= \begin{bmatrix} P_{co}^{h} & \sqrt{2}R_{cx}^{h} & R_{co} \\ \sqrt{2}(R_{cx}^{h})^{*} & 2P_{cx} & \sqrt{2}(R_{cx}^{v})^{*} \\ (R_{co})^{*} & \sqrt{2}R_{cx}^{v} & P_{co}^{v} \end{bmatrix}$$

P^h_{co} = co-polar power received in the H channel (e.g., Z_h)

P^v_{co} = co-polar power received in the V channel (e.g., Z_v)

P_{cx} = cross-polar power (e.g., LDR)

$R_{co,cx}$ = correlation terms (e.g., R_{co} or R_{co}^* used in computation of ρ_{hv} and the $\arg(R_{co})^*$ contains phase information due to propagation and backscatter differential phase, and the Doppler velocity)

* = complex conjugate

From these we define:

$$\textbf{ZDR} = 10\mathrm{Log}\left(\frac{P^h_{co}}{P^v_{co}}\right) = 10\mathrm{Log}\frac{|S_{hh}|^2}{|S_{vv}|^2}$$

Differential Reflectivity (dB)

$$\textbf{LDR} = 10\mathrm{Log}\left(\frac{P_{cx}}{P^h_{co}}\right) = 10\mathrm{Log}\frac{|S_{vh}|^2}{|S_{hh}|^2}$$

Linear Depolarization Ratio (dB)

$$|\rho_{\mathbf{co}}| = \frac{|R_{co}{}^*|}{(P^h_{co}P^v_{co})^{1/2}} = \frac{|S_{vv}||S_{hh}|}{(|S_{hh}|^2|S_{vv}|^2)^{1/2}}$$

Co-polar correlation coefficient (ρ_{hv})

And for the phase variables:

$$\rho_{co} = \frac{|S_{vv}|\,|S_{hh}|\, e^{-j(\delta_{co}+\Phi_{dp})}}{(|S_{hh}|^2 |S_{vv}|^2)^{½}}$$

$$\arg(\rho_{co}) = \Psi_{dp} = \delta_{co} + \Phi_{dp}$$

Where,

Ψ_{dp}= Differential phase (Degrees)

$\mathbf{\Phi_{dp}}$ = Propagation differential phase (Degrees)

δ_{co} = backscatter differential phase (Degrees) [Mie effect]

Note:

σ_{ZDR} will increase if $|\rho_{co}|$ decreases (ρ_{co} can also be used as an overall indicator of noise in the system; should be very close to 1 in rain).

For Rayleigh conditions, δ_{co} is small and the measured differential phase is essentially the Φ_{dp}.

Doppler velocity is estimated from the sum of the total phase shift of consecutive pulses (the argument of the autocorrelation between those pulses; see Sec. 6.4 in BC and also eq. 5.208).

Finally, we can define the **specific differential phase (K_{dp})** as the range derivative of differential propagation phase:

$$\mathbf{K_{dp}} = ½ \frac{d\mathbf{\Phi}_{dp}}{dr}$$

In practice, $\mathbf{\Phi_{dp}}$ is filtered prior to computing $\mathbf{K_{dp}}$