ATMO 489/689
Radar Meteorology

Laboratory #7
11/14/05 (Monday section) and 11/15/05 (Tuesday section)

Attenuation, Rain Rate (R) and z-R relations

Due: By 5 PM on Wednesday, 11/23/05

Reading

Textbook/Rinehart (2004), Chapter 8

Background on Rain Rate and z-R Relations

In Laboratory #5, we derived an expression for the **radar reflectivity factor** (z) as a function of the rain drop diameter, D, and size distribution, N(D), which is the number of rain drops of size D per unit volume. We found that radar reflectivity factor is the 6^{th} moment of the rain drop size distribution for Rayleigh scattering conditions.

$$z = \int_0^\infty N(D)\, D^6\, dD \approx \sum_{i=1}^{M} N(D_i)\, D_i^{\,6} \qquad (1)$$

For most precipitation wavelengths (e.g., X-band to S-band), rainfall is mostly to entirely in the Rayleigh scattering regime. Next, we defined z ($mm^6\ m^{-3}$) specifically for drop size distribution data collected by a Joss-Waldvogel (J-W) disdrometer.

$$z = \frac{1}{T \bullet A} \sum_{i=1}^{M} \frac{C_i \bullet D_i^6}{v_i(D_i)} \qquad [mm^6\ m^{-3}] \qquad (2)$$

where i is the bin (or channel) number, C_i is the concentration (i.e., #) of drops per D_i bin, M is the total number of channels, D_i is the i^{th} diameter size (mm), T is the sampling time (seconds), A is the sample area (m^2), and v_i is the terminal fall speed of rain drops ($m\ s^{-1}$). For the J-W disdrometer, M = 20, T = 60 seconds and $A = 5 \times 10^{-3}\ m^2$.

Rain Rate (R) is a measure of the intensity of rainfall and is typically expressed as the equivalent depth (or length) of liquid water accumulated on the ground per unit time (e.g., millimeters per hour or $mm\ h^{-1}$). Given the definition above, we can derive an expression for rain rate in still air (i.e., no updraft or downdraft present) of the form

$$R = \int_0^{\infty} N(D)\, V(D)\, v(D)\, dD \tag{3}$$

where V(D) and ν(D) are the volume and terminal fall speed, respectively, of a spherical rain drop of diameter D. For a sphere, V(D) = (π/6)*D^3 such that

$$R = \frac{\pi}{6}\int_0^{\infty} N(D)\; D^3\, v(D)\, dD \approx \; \frac{\pi}{6}\sum_{i=1}^{M} N(D)\; D^3\, v(D) \tag{4}$$

By analogy with the expression for reflectivity factor in (2), we can convert (4) into an expression that is valid for drop size distribution data collected by a Joss-Waldvogel (J-W) disdrometer.

$$R = \frac{\pi}{6 \bullet T \bullet A}\sum_{i=1}^{M} \frac{C_i \bullet D_i^3 \bullet v_i(D_i)}{v_i(D_i)} = \; \frac{\pi}{6 \bullet T \bullet A}\sum_{i=1}^{M} C_i \bullet D_i^3 \tag{5}$$

Eqn. (5) currently has units of $mm^3\ s^{-1}\ m^{-2}$. After multiplying (5) by the appropriate conversion factor to get the conventional units for R of mm h^{-1}, we obtain

$$R = \frac{6\pi \times 10^{-4}}{T \bullet A}\sum_{i=1}^{M} C_i \bullet D_i^3 \qquad [\text{mm h}^{-1}] \tag{6}$$

The purpose of question 1 and 2 in this lab is to explore the calculation of rainfall rate and to understand its relationship to radar reflectivity factor. Ultimately, we want to understand how radars can use low-level PPI observations of z ($mm^6\ m^{-3}$) to estimate near surface R (mm h^{-1}) using a so called z-R relationship of the form

$$z = a\, R^b \tag{7}$$

where a, b are assumed constant for a given situation.

Questions (100 points)

1. (55 points) Derive rain rates and a custom z-R relationship from the TAMU J-W disdrometer data taken in College Station on December 22, 2004 from 1327 UTC to 1352 UTC, which was provided earlier in Table 1 of Lab #5 (see also the associated space-delimited data file (disdrom_data.prn) or the tab-delimited data file (disdrom_data.txt) available on the class web page in the lab section).

a. (25 points) Using the data in Table 1 and the definition in equation [6], calculate and plot a time series of rain rate (R, mm h^{-1}). You are encouraged to use either a computer programming language or a spreadsheet program of your choice to solve this problem. Several options are available on the LINUX PC's in Rm 1201.

b. (15 points) Using pairs of radar reflectivity factor (z, $mm^6\ m^{-3}$) (from Lab #5) and the rain rate (R, mm h^{-1}) (from a) above) for each of the 26 1-minute periods from 1327 to 1352 UTC, make a scatter plot of R (ordinate) vs. z (abscissa). Next, derive a z-R power-law relation specifically for this rainfall event in the form given by (7) above. ***NOTE:*** By convention, radar meteorologists use (7) above even though z is the independent (i.e., measured) value and R is the dependent (i.e., calculated) value. In order to derive an unbiased z-R equation, you should use a computer programming language or a spreadsheet program of your choice to fit a power law equation to the R-z data of the form $R = cz^d$. Provide a plot of the fitted curve on top of the R-z points and the coefficients c and d. Next, invert $R = cz^d$ to obtain the more conventional $z = aR^b$ and provide the coefficients a and b.

c. (5 points) As a radar meteorologist, you might use your regressed z-R relation from b) with measured reflectivity (z) from nearby radars (e.g., KHGX, KGRK, KEWX, KLCH) to estimate rainfall over a much larger area of southeastern Texas (see Fig. 3 from Lab #5) than could be practically covered by rain gauges. Visually compare the regressed power law equation and plotted R-z points in b). Do all of the R-z points fall on the regressed curve? What is the implication for the radar estimation of rainfall using a z-R technique?

d. (10 points) Since the DSD, N(D), can vary significantly from one rain even to another, so can the associated z-R relation. Of course, radar meteorologists do not typically have disdrometer measurements of the drop size distribution (DSD) for each rainfall event. As a result, we must assume a z-R relation in order to compute rainfall from low-level PPI scans of z. Some typical relationships are

1. $z = 200\ R^{1.6}$ Marshall-Palmer z-R
2. $z = 300\ R^{1.4}$ WSR-88D conventional z-R
3. $z = 250\ R^{1.2}$ WSR-88D tropical z-R

On a single figure, make a semi-log plot of R (ordinate) vs. z (logarithmic abscissa) for the three common z-R relations above along with your derived R-z relation from b) above for a range of z ($10^0 \le z \le 10^5\ mm^6\ m^{-3}$). How do the three common R-z curves compare to each other? How does your derived R-z curve compare to the three common relations above? What is the implication for the radar estimation of rainfall using a z-R technique?

2. (20 points) As discussed in lab #5, the terminal fall speed can be expressed in terms of a power law equation of the form $v(D) = \alpha D^{\beta}$ where α and β are constants. As discussed in class earlier, the drop size distribution, N(D), can be expressed in the form $N(D) = N_0 \exp(-\lambda D)$ where N_0 and $-\lambda$ are the intercept and slope of the inverse-exponential relationship on a semi-log plot (e.g., see Fig. 8.2 in Rinehart 2004). Using the *definite* integral definition of z and R from (1) and (4), respectively, to derive an expression of the form $z = aR^b$ where a and b are a function of N_0, α, and/or β. Observations have shown that N_0 varies widely from one rain event to another and even during different stages of the same rain event while α and β do not vary much. What is the implication for "a" and "b" in the $z = aR^b$ relation and the radar estimation of rainfall using the z-R technique?

Note: You may find the following definite integral useful for this problem

$$\int_0^{\infty} x^{\omega-1} e^{-\mu x} \, dx = \frac{\Gamma(\omega)}{\mu^{\omega}}$$

where $\Gamma(\omega) = (\omega - 1)!$

3. (25 points) Assume that the attenuation coefficient for rain (K_r, dB km^{-1}) at S-band can be expressed by, $K_r = 8.0 \times 10^{-4} \bullet R$, where R is rain rate (mm h^{-1}). For a particular cloud, the profile of rain rate as a function of range (r) is

$$R = 50 - (r-100)^2/8 \quad [mm\, h^{-1}] \quad for \quad 80 \le r \le 120 \; km$$
$$R = 0 \qquad otherwise$$

a. (10 points) Calculate the two-way attenuation loss associated with rain (L_r, dB) in this cloud at a range of 100 km.
b. (4 points) The attenuation coefficient for atmospheric gas (K_g) at S-band is $K_g = 0.008$ dB km^{-1}. Calculate the two-way attenuation loss associated with atmospheric gas (L_g, dB) at a range of 100 km. Assuming all other attenuation losses can be neglected, what is the total attenuation loss ($L \approx L_g + L_r$, dB)?
c. (4 points) Assume rain rate (R, mm h^{-1}) and radar reflectivity factor (z, mm^6 m^{-3}) are related by the following power law equation: $z = 300 \bullet R^{1.4}$. Given the equation for rain rate (R) above, what is the *actual* reflectivity factor z (mm^6 m^{-3}) and Z (dBZ) at r = 100 km?
d. (4 points) If the total attenuation loss (L) were ignored in the radar range equation, what Z (dBZ) and z (mm^6 m^{-3}) would be *measured* at r = 100 km by a *perfect* S-band radar (all else being equal)? (Hint: you don't need to solve the radar range equation!)
e. (3 points) Using the z-R relationship from part c), estimate the rainfall rate (R, mm h^{-1}) from the *measured* reflectivity factor (z) in part d) (i.e., ignoring attenuation losses). What would the error in the estimated rain rate be at 100 km range that would result from ignoring attenuation losses?