ATMO 489/689
Radar Meteorology

Laboratory #4
10/10/05 (Monday section) and 10/11/05 (Tuesday section)

The Complex Index of Refraction
Or, The Right Stuff

Due: By beginning of next lab session

Reading:

Ch. 3 p. 54; Ch. 4 p. 73; Ch. 5 p. 92-93 in Rinehart (2004)

Background:

The refractive index of electromagnetic radiation (e.g., radar waves!) is actually a complex number, m, typically given by

$$m = n - ik$$

where $i = \sqrt{-1}$, n is the real component discussed earlier in lecture (and Chapter 3 of the textbook), and k is the imaginary component. For notational simplicity, sometimes we will refer to m = (n,k).

The imaginary component of the refractive index, k, is related to the absorption coefficient of the medium. For a perfect (i.e., lossless) dielectric (or non-conductor), k is equal to zero. A material with relatively large k has a relatively large absorption loss of energy, which is essentially converted to internal heat energy. Later when we discuss the "attenuation" of radar waves, we will learn that a medium with relatively large k causes relatively large attenuation of radar waves or loss of the radar signal as it propagates through the medium.

In laboratory 3, we discussed a definition of the Rayleigh backscattering cross-sectional area (σ) that was a function of several variables, including $|K|^2$, which is a function of m. As a matter of fact, the parameter K, sometimes called the "dielectric function," is given by[1]

$$K = \frac{m^2 - 1}{m^2 + 2}$$

A word of warning before calculating $|K|^2$ - review your basic complex algebra!

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[1] Note that the equation for K on p. 92 of Rinehart (2004) has an error. The m in the numerator should be squared.

Objective:

The objective of this lab is to further define the complex index of refraction; its dependence on temperature, wavelength, and hydrometeor composition; and its relevance to and implications for backscattering in radar meteorology.

Tools:

We will use another IDL program, **ray.pro**, which is available on the class web page as a link in the lab section. The program calculates the complex index of refraction (n,k) of water and ice using empirical equations compiled by Ray (1972) as a function of 1) temperature (T, °C), 2) wavelength (λ, cm), and 3) composition or thermodynamic phase (i.e., water or ice). Phase (ph) is indicated by a binary integer where ph=0 for water and ph=1 for ice.

The equations in Ray (1972) and hence the program are valid for the following temperature ranges for ice: -20C to 0C; and for water: -20C to 50C. All radar bands and associated wavelengths given in Rinehart (2004) Table 3.1 are valid.

Running the ray.pro program on a LINUX PC in Rm 1201,

```
idl
>ray, t, λ, ph
> exit
```

For example, we can run the program to calculate the complex index of refraction for T=0°C, λ=3.21cm (X-band). The IDL command would be:

```
>ray, 0.0, 3.21, 0
```

The IDL program would output n, k.

```
% Compiled module: RAY.
     7.2620630      2.8538182
% Stop encountered: RAY
```

Here n=7.26 and k=2.85, which are similar to values we used from Battan (1973) in Laboratory #3.

To quit or exit the program,

```
>exit
```

Questions (50 points):

1. (5 points) Using provided IDL program ray.pro, determine the complex index of refraction (n,k) for **water** at S-band (10 cm), K_u-band (1.8 cm), and W-band (0.3 cm) at T = 20°, 10°, 0° C. Put your results in tabular format.

2. (5 points) Using provided IDL program ray.pro, determine the complex index of refraction (n,k) for **ice** at S-band (10 cm), K_u-band (1.8 cm), and W-band (0.3 cm) at T = 0°, -10°, and -20° C. Put your results in tabular format.

3. (8 points) Using your results in parts 1) and 2) above, discuss the dependence of the complex index of refraction on wavelength, temperature, and composition (i.e., water vs. ice).

4. (8 points) Using mie_ray_plot.pro from laboratory 3, generate plots of the 1) normalized backscattering cross-section, $\sigma/(\pi r^2)$, and 2) the absolute error associated with assuming Rayleigh scattering, $(\sigma_{Mie} - \sigma_{Ray})/ \sigma_{Mie} * 100\%$ versus the size parameter $\alpha = 2\pi r/\lambda$ for S-band (10 cm) for **ice** spheres at T=0°C. Discuss how your new results compare with your laboratory #3 results for water spheres.

5. (8 points) If you define Rayleigh scattering region as the maximum diameter (D_{ray}) for which absolute error < 25%, what would D_{ray} be for **ice** spheres (e.g., hail) at T=0°C for S-band? Compare and contrast your new result with your laboratory #3 result for water spheres. Is the definition of the "Rayleigh Scattering Regime" the same or different for ice and water spheres? Discuss.

6. (8 points) Using results from parts 1) and 2) above, calculate $|K|^2$ for water and ice at S-band and W-band, assuming T=0°C. What would be the consequence of using the S-band $|K|^2$ in the calculation of backscattering cross-sectional area (σ) at W-band?

7. (8 points) Assuming S-band (10 cm) and Rayleigh scattering, hand calculate (i.e., *not* with IDL program but calculator fine) σ of a **water** sphere of diameter D = 5 mm (show work). Repeat your calculation for an **ice** sphere of the same size for the same wavelength. Since the three-dimensional distribution of the thermodynamic phase of hydrometeors is not generally known within a precipitating cloud, radar meteorologists typically assume that every part of the storm being scanned is made up of water. What is the implication of this assumption for Rayleigh backscattering? In other words, what error in the calculation of σ would be caused by assuming water hydrometeors when they are really ice?