

ATMO 489
Radar Meteorology

Laboratory #9
11/13/06

Attenuation, Rain Rate (R) and z-R relations

Due: Monday, 11/27/06 at the beginning of lab.

Reading

Rinehart (2004), Chapter 8

Background on Rain Rate and z-R Relations

In Laboratory #7, we derived an expression for the **radar reflectivity factor** (z) as a function of the rain drop diameter, D , and size distribution, $N(D)$, which is the number of rain drops of size D per unit volume. We found that radar reflectivity factor is the 6th moment of the rain drop size distribution for Rayleigh scattering conditions.

$$z = \int_0^{\infty} N(D) D^6 dD \approx \sum_{i=1}^M N(D_i) D_i^6 \quad (1)$$

For most precipitation wavelengths (e.g., X-band to S-band), rainfall is mostly or entirely in the Rayleigh scattering regime. Next, we defined z ($\text{mm}^6 \text{m}^{-3}$) specifically for drop size distribution data collected by a Joss-Waldvogel (J-W) disdrometer.

$$z = \frac{1}{T \bullet A} \sum_{i=1}^M \frac{C_i \bullet D_i^6}{v_i(D_i)} \quad [\text{mm}^6 \text{m}^{-3}] \quad (2)$$

where i is the bin (or channel) number, C_i is the concentration (i.e., #) of drops per D_i bin, M is the total number of channels, D_i is the i^{th} diameter size (mm), T is the sampling time (seconds), A is the sample area (m^2), and v_i is the terminal fall speed of rain drops (m s^{-1}). For our J-W disdrometer, $M = 20$, $T = 10$ seconds and $A = 5 \times 10^{-3} \text{m}^2$.

Rain Rate (R) is a measure of the intensity of rainfall and is typically expressed as the equivalent depth (or length) of liquid water accumulated on the ground per unit time (e.g., millimeters per hour or mm h^{-1}). Given the definition above, we can derive an expression for rain rate in still air (i.e., no updraft or downdraft present) of the form

$$R = \int_0^{\infty} N(D) V(D) v(D) dD \quad (3)$$

where $V(D)$ and $v(D)$ are the volume and terminal fall speed, respectively, of a spherical rain drop of diameter D . For a sphere, $V(D) = (\pi/6)*D^3$ such that

$$R = \frac{\pi}{6} \int_0^{\infty} N(D) D^3 v(D) dD \approx \frac{\pi}{6} \sum_{i=1}^M N(D) D^3 v(D) \quad (4)$$

By analogy with the expression for reflectivity factor in (2), we can convert (4) into an expression that is valid for drop size distribution data collected by a Joss-Waldvogel (J-W) disdrometer.

$$R = \frac{\pi}{6 \bullet T \bullet A} \sum_{i=1}^M \frac{C_i \bullet D_i^3 \bullet v_i(D_i)}{v_i(D_i)} = \frac{\pi}{6 \bullet T \bullet A} \sum_{i=1}^M C_i \bullet D_i^3 \quad (5)$$

Eqn. (5) currently has units of $\text{mm}^3 \text{s}^{-1} \text{m}^{-2}$. After multiplying (5) by the appropriate conversion factor to get the conventional units for R of mm h^{-1} , we obtain

$$R = \frac{6\pi \times 10^{-4}}{T \bullet A} \sum_{i=1}^M C_i \bullet D_i^3 \quad [\text{mm h}^{-1}] \quad (6)$$

The purpose of question 1 and 2 in this lab is to explore the calculation of rainfall rate and to understand its relationship to radar reflectivity factor. Ultimately, we want to understand how radars can use low-level PPI observations of z ($\text{mm}^6 \text{m}^{-3}$) to estimate near surface R (mm h^{-1}) using a so called z - R relationship of the form

$$z = a R^b \quad (7)$$

where a , b are assumed constant for a given situation.

Questions (100 points)

1. (75 points) Derive rain rates and custom z-R relations from the TAMU J-W disdrometer data taken in a mesoscale convective system (MCS) over College Station from 15:13:00 to 15:34:00 CST on October 31, 2005. Recall that the J-W data was provided earlier in Table 1 of Lab #7 and the associated data files available on the class web page.

a. (25 points) Using the data in Table 1 and the definition in equation [6], calculate and plot a time series of rain rate (R , mm h^{-1}). You are encouraged to use either a computer programming language or a spreadsheet program of your choice to solve this problem. Several options are available on the LINUX PC's in Rm 1201.

b. (15 points) Using pairs of radar reflectivity factor (z , $\text{mm}^6 \text{m}^{-3}$) (from Lab #7) and the rain rate (R , mm h^{-1}) (from part a above) for each of the 10-second periods from 15:13:00 CST to 15:23:00 CST, create a scatter plot of R (ordinate) vs. z (abscissa). Next, derive a z-R power-law relation specifically for this rainfall event in the form given by (7) above. **NOTE:** By convention, radar meteorologists use Equation (7) above even though z is the independent (i.e., measured) value and R is the dependent (i.e., calculated) value. In order to derive an unbiased z-R equation, you should use a computer programming language or a spreadsheet program of your choice to fit a power law equation to the R-z data of the form $R = cz^d$. Provide a plot of the fitted curve on top of the R-z points and the coefficients c and d . Next, invert $R = cz^d$ to obtain the more conventional $z = aR^b$ and provide the coefficients a and b .

c. (15 points) Repeat part b above for the time period between 15:23:00 CST and 15:34:00 CST.

d. (5 points) Based on the z (Lab #7) and R time series, the convective line of the MCS passed over the J-W disdrometer from about 15:13-15:23 CST while the trailing stratiform region passed over from about 15:23-15:34 CST. Past research has shown that convective and stratiform region z-R relations are fundamentally different because of different primary cloud microphysical processes (e.g., riming/graupel vs. aggregation/deposition/snow, respectively). How do your convective and stratiform z-R relations compare?

c. (5 points) As a radar meteorologist, you might use your regressed z-R relations from b) and c) with measured reflectivity (z) from nearby radars (e.g., KHGX, KGRK, KEWX, KLCH) to estimate rainfall over a much larger area of southeastern Texas than could be practically covered by rain gauges. Visually compare the regressed power law equation and plotted R-z points in b) and c). Do all of the R-z points fall on the regressed curve? What is the implication for the radar estimation of rainfall using a z-R technique?

d. (10 points) Since the DSD, $N(D)$, can vary significantly from one rain event to another, so can the associated z-R relation. Of course, radar meteorologists do not typically have disdrometer measurements of the drop size distribution (DSD) for each rainfall event. As a result, we must assume a z-R relation in order to compute rainfall from low-level PPI scans of z . Some typical z-R relationships are

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|----------------------|---|
| 1. $z = 200 R^{1.6}$ | Marshall-Palmer z-R (light rain/stratiform) |
| 2. $z = 300 R^{1.4}$ | WSR-88D conventional convective z-R |
| 3. $z = 250 R^{1.2}$ | WSR-88D tropical convective z-R |

On a single figure, make a semi-log plot of R (ordinate) vs. z (logarithmic abscissa) for the three common z-R relations above along with your derived convective and stratiform R-z relations from b) and c) above for a range of z ($10^0 \leq z \leq 10^6 \text{ mm}^6 \text{ m}^{-3}$). How do the three common R-z curves compare to each other? How do your derived R-z curves compare to the three common relations above? What are the implications for the radar estimation of rainfall using a z-R technique in this and other MCS's?

2. (25 points) Assume that the attenuation coefficient for rain (K_r , dB km^{-1}) at S-band can be expressed by, $K_r = 8.0 \times 10^{-4} \bullet R$, where R is rain rate (mm h^{-1}). For a particular cloud, the profile of rain rate as a function of range (r) is

$$R = 50 - (r - 100)^2 / 8 \quad [\text{mm h}^{-1}] \quad \text{for } 80 \leq r \leq 120 \text{ km}$$

$$R = 0 \quad \text{otherwise}$$

- (10 points) Calculate the two-way attenuation loss associated with rain (L_r , dB) in this cloud at a range of 100 km.
- (4 points) The attenuation coefficient for atmospheric gas (K_g) at S-band is $K_g = 0.008 \text{ dB km}^{-1}$. Calculate the two-way attenuation loss associated with atmospheric gas (L_g , dB) at a range of 100 km. Assuming all other attenuation losses can be neglected, what is the total attenuation loss ($L \approx L_g + L_r$, dB)?
- (4 points) Assume rain rate (R, mm h^{-1}) and radar reflectivity factor (z, $\text{mm}^6 \text{ m}^{-3}$) are related by the following power law equation: $z = 300 \bullet R^{1.4}$. Given the equation for rain rate (R) above, what is the *actual* reflectivity factor z ($\text{mm}^6 \text{ m}^{-3}$) and Z (dBZ) at r = 100 km?
- (4 points) If the total attenuation loss (L) were ignored in the radar range equation, what Z (dBZ) and z ($\text{mm}^6 \text{ m}^{-3}$) would be *measured* at r = 100 km by a *perfect* S-band radar (all else being equal)? (Hint: you don't need to solve the radar range equation!)
- (3 points) Using the z-R relationship from part c), estimate the rainfall rate (R, mm h^{-1}) from the *measured* reflectivity factor (z) in part d) (i.e., ignoring attenuation losses). What would the error in the estimated rain rate be at 100 km range that would result from ignoring attenuation losses?