ATMO 489: Radar Meteorology

Laboratory #5
10/9/06

RADAR SIZE
Or, to Mie or not to Mie

Due: By beginning of next lab session

Reading:

Chapter 4 Rinehart (2004) (p. 73 – 75)

Background:

Besides for simply determining the range to a target, weather radars are capable of measuring the received power (P_r) returned or "backscattered" from a target at some range (r). If the radar system is calibrated, then it is theoretically possible to relate P_r to physical properties of the target. As shown in lecture, this returned power is a function of the "radar size" in addition to range to the target and several radar parameters such as transmitted power (P_t), gain (g), and wavelength (λ). In radar meteorology, the "radar size" is typically defined by the **backscattering cross-sectional area** (σ). For a point target, we will show that

$$P_r \propto \sigma \qquad [1]$$

In other words, the received power increases (or decreases) in direct proportion to a larger (or smaller) "radar size" or backscattering cross-sectional area. The back scattering cross-sectional area is a function of target properties such as the size, shape, and kind of matter (i.e., complex refractive index; m = n – ik, where n is the real and k is the imaginary component) and of radar properties such as the wavelength.

When possible, radar meteorologists use scattering theory for relatively simple shapes such as spheres, spheroids, columns etc, which represent many hydrometeors fairly well, to calculate σ as a function of size, refractive index, and wavelength. The simplest assumption for shape is that of a sphere. As a result, it is the one most often used by radar meteorologists.

When a *sphere is large compared to the wavelength* (i.e., diameter, $D > 10\lambda$), the back-scattering cross-sectional area approaches the geometric area or

$$\sigma \approx \pi r^2 \qquad [2]$$

where r is the radius ($r = D/2$) of the sphere.

When the size (i.e., diameter, D) of a *sphere is small compared to the wavelength* (i.e., D < 0.1λ), the sphere is in what is called the *Rayleigh scattering* region. In the Rayleigh region, σ is proportional to the sixth power of the diameter. From Rayleigh scattering, we can easily calculate back-scattering cross-sectional area and hence returned power from spheres in this scattering region. For a sphere, the expression for σ is given by

$$\sigma = \frac{\pi^5 |K|^2 D^6}{\lambda^4} \qquad [3]$$

where $|K|^2$ is the dielectric function, which is a function of the complex index of refraction of the material. We will study the dielectric function and refractive index of water and ice hydrometeors in more detail in a later laboratory. Many hydrometeors are small compared to λ so the Rayleigh scattering region is often assumed for meteorological targets. Some targets, especially non-meteorological ones such as ground targets (e.g., buildings) and aircraft are large compared to the wavelength. However, there is a vast intermediate region where neither assumption is valid.

In 1908, *Mie* determined analytical expressions necessary to calculate σ of a sphere of *any diameter* (including the intermediate ones), given the wavelength and refractive index. From *Mie theory*, it can be shown that

$$\sigma = \frac{\pi r^2}{\alpha^2} \left| \sum_{n=1}^{\infty} (-1)^n (2n+1)(a_n - b_n) \right|^2 \qquad [4]$$

where r is the hydrometeor radius, α is the size parameter where α = 2πr/λ, the quantities a_n and b_n are coefficients of the backscattered field, and n stand for the number of terms in the expansions of a_n and b_n. The term a_n refers to the scattering arising from the induced magnetic dipoles, quadrapoles, etc and the term b_n refers to the electric dipoles, quadrapoles, etc. The terms a_n and b_n can be expressed in terms of spherical Bessel and Hankel functions of the second kind with arguments of α and the complex refractive index, m = n – ik, where n is the real and k is the imaginary component.

Fortunately for us, these mathematical functions have already been programmed into an IDL (Interactive Data Language) program using Mie theory for calculating σ so that we don't have to worry about their form at all!

Objective:

The purpose of this lab is to explore values for the backscattering cross-sectional area, σ, using both Rayleigh and Mie theories for spherical water hydrometeors at different wavelengths using a tool written in IDL (Interactive Data Language). In particular, we want to understand the meaning of the "Rayleigh region," more specifically where this region is, and what sort of error is associated with making the "Rayleigh approximation" (i.e., the target sphere is small compared to the wavelength). In a future lab, we will explore in more detail the effect of the refractive index, which is a function of wavelength, temperature, and the material of the hydrometeor (e.g., ice vs. water).

Tools:

We will use a couple of programs in IDL that are available on the class web page as links in the lab section

Program	*Description*	*Output*
bhmie.pro	Calculates σ using Mie theory as found in Bohren and Huffman (1983)	NA
mie_ray_plot.pro	Calls bhmie.pro and calculates σ using Rayleigh theory. Makes plots of the normalized backscattering cross-section $[\sigma/(\pi r^2)]$ for both Mie and Rayleigh theory, the absolute error term in assuming Rayleigh theory (i.e., $(\sigma_{Mie} - \sigma_{Ray})/\sigma_{Mie} * 100\%$, and the ratio of $\sigma_{Mie}/\sigma_{Ray}$	*Postscript files:* mie_ovr_ray.ps, mie_ray_error.ps, nsigma_mie_ray.ps. *ASCII text files:* mie_ray.dat, mie_ray_error.dat, norm_mie_ray.dat

Running the mie_ray_plot.pro program

```
idl
>mie_ray_plot, lam, x1, x2
> exit
```

Where lam = wavelength (λ, cm)
x1 = real component of the refractive index (or n)
x2 = imaginary component of the refractive index (or k)

Questions (40 points):

1. (10 points) Using mie_ray_plot.pro, generate plots of the 1) normalized backscattering cross-section, $\sigma/(\pi r^2)$, 2) the ratio of the $\sigma_{Mie}/\sigma_{Ray}$, and 3) the absolute error associated with assuming Rayleigh scattering, $(\sigma_{Mie} - \sigma_{Ray})/\sigma_{Mie} * 100\%$ versus the size parameter $\alpha = 2\pi r/\lambda$ for both S-band (10 cm) and X-band (3.21 cm) for water spheres at T=0°C

Refractive index for water at T=0°C

Radar Band	Real component of refractive index (x1 or n)	Imaginary component of the refractive index (x2 or k)
S-band, 10 cm	8.99	1.47
X-band, 3.21 cm	7.14	2.89

From Battan (1973)

2. (5 points) What is the maximum error in the back-scattering cross section associated with assuming Rayleigh scattering for $D < 0.1\ \lambda$ for water spheres (i.e., rain) at T = 0° C for S-band and X-band ?

3. (10 points) If you define the Rayleigh scattering region as the maximum diameter (D_{ray}) for which the absolute error in the back-scattering cross-section < 25 %, what would D_{ray} be for water spheres (i.e., rain) at T = 0° C for S-band and X-band ? What about < 10 % absolute error?

4. (5 points) At what size parameter (α) and diameter (D, mm) do the Rayleigh approximation for the back-scattering cross-section deviate from Mie by a factor of 3 dB or more for S-band and X-band, assuming rain at 0° C?

5. (10 points) Based on what you have learned, is the Rayleigh scattering approximation for the back-scattering cross-sectional area sufficiently accurate for use in the radar range equation for all precipitation hydrometeors at S-band? What about X-band? Give examples of typical hydrometeors and their sizes (i.e., spherical diameters) and whether you believe that the Rayleigh approximation would be sufficiently accurate for each particle at each wavelength. (Note: Assume all of your hydrometeors are water or sufficiently water coated to be effective water spheres at 0° C.)