Read this first!

The rules for this exam are as follows:

- This is a timed exam: you must finish the exam within 90 minutes of starting (which should be plenty of time). Please do not look at the exam questions until you are ready to begin.
- The exam is open book, but it is not collaborative. You must complete the exam entirely by yourself.
- You must hand in the the exam at (or before) the beginning of class on Wednesday, April 23.
- Please sign the following to indicate your acceptance of these rules.

By signing below, I indicate that I have read the rules given above and agree to follow them in good faith. I also acknowledge that a failure to follow these rules will be considered an honor code violation.

Sign and date: ____

Please complete the following to indicate your time spent on the exam:

Time exam started: ______________________________

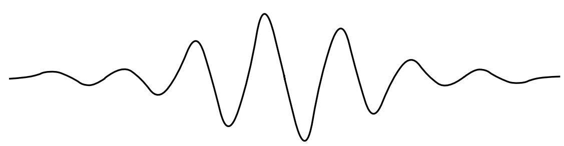
Time exam completed: ___________________________

Two on the shorter side (20 pts each):

1. *Rayleigh waves* are a type of vorticity-driven motion that propagates along an interface between two shear layers. The dispersion relation for Rayleigh waves is

$$\omega = -\frac{S}{2}$$

where the constant S is related to the difference in shear between the two layers. Given this relation, would you expect the wave pattern illustrated below to maintain its shape over time? **Briefly** explain.



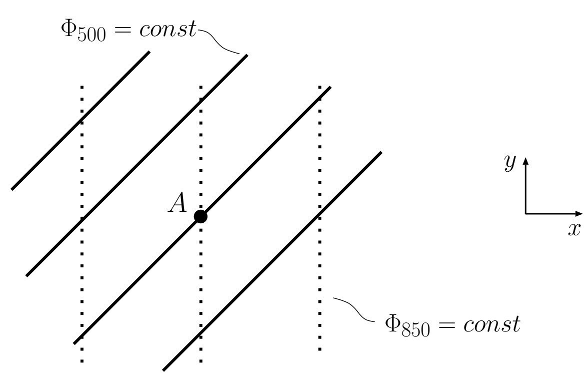
2. Using the diagnostic pressure equation, explain why a tornado typically has low pressure at the center.

One a bit longer (25 pts):

3. Suppose that the geopotential distributions for the 850 hPa and 500 hPa pressure surfaces are specified by

$$\Phi_{850}(x, y) = \Phi_A + \kappa x \qquad \text{and}$$
$$\Phi_{500}(x, y) = \Phi_B + \kappa x - \kappa y$$

where Φ_A and Φ_B are constants and where κ is assumed positive. Let the Coriolis parameter be given by f (also positive).



In which direction does the temperature of the air between the two pressure levels increase most rapidly (i.e., which direction has the largest temperature gradient)? Justify your answer.

And one more on the back.....

And one long one (35 pts):

4. Consider a storm that develops in a background shear profile described by

$$\mathbf{U}(z) = \Lambda z\, \hat{\mathbf{y}} = (0,\, \Lambda z,\, 0) \tag{1}$$

where Λ is a positive constant.

(a) Compute the vorticity vector $\boldsymbol{\omega} = (\xi,\, \eta,\, \zeta)$ corresponding to (1) and sketch the associated vortex lines over the domain $-2 \le x \le 2$ and $-2 \le y \le 2$ in the xy plane.

(b) Suppose that on top of this shear profile we superimpose an updraft, so that at time $t = 0$ the total wind is described by

$$\mathbf{u} = \mathbf{U} + w_0\, e^{-x^2-y^2}\, \hat{\mathbf{z}} = \left(0,\, \Lambda z,\, w_0\, e^{-x^2-y^2}\right) \tag{2}$$

where w_0 is the maximum updraft strength. Indicate the center of this updraft on your sketch from (a).

(c) Using the flow field defined by (2), compute the rate of vertical vorticity production $D\zeta/Dt$ (at $t = 0$).

(d) Find the x, y coordinates for the locations of maximum (most positive) and minimum (most negative) $D\zeta/Dt$ as diagnosed from (c). (*Hint:* Given a function, how do you find a maximum or minimum?) Indicate these locations on your sketch, using + for the maximum and − for the minimum.

(e) After some time the vortex lines sketched in (a) will be deformed by the flow. Sketch the expected vortex line pattern as seen in vertical cross section—specifically, consider the cross section that passes through the maximum and minimum $D\zeta/Dt$ locations as diagnosed in (d).