1

We're given

$$\Phi = \Phi_0(p) - U f_0 y \cos\left(\frac{\pi}{3}\frac{p}{p_0} + \frac{\pi}{6}\right) + \frac{U}{2k} f_0 \sin\left(kx + \pi\frac{p_0 - p}{p_0}\right)$$

where $p_0 = 1000\,\text{hPa}$

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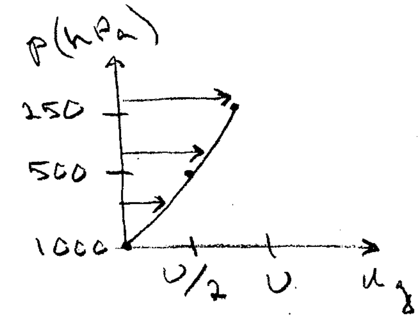
a

Our geostrophic wind follows from

$$u_g = -\frac{1}{f_0}\frac{\partial \Phi}{\partial y} = U\cos\left(\frac{\pi}{3}\frac{p}{p_0} + \frac{\pi}{6}\right)$$

$$v_g = \frac{1}{f_0}\frac{\partial \Phi}{\partial x} = \frac{U}{2}\cos\left(kx + \pi\frac{p_0 - p}{p_0}\right)$$

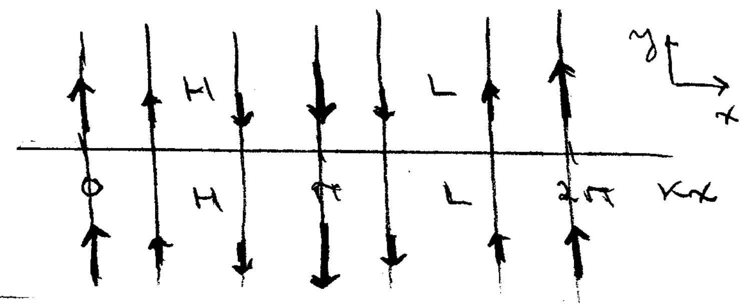


At $p = 1000\,\text{hPa}$ we have

$$\Phi = \Phi_0\big|_{1000} + \frac{U}{2k} f_0 \sin(kx)$$

$$u_g = 0$$

$$v_g = \frac{U}{2}\cos(kx)$$

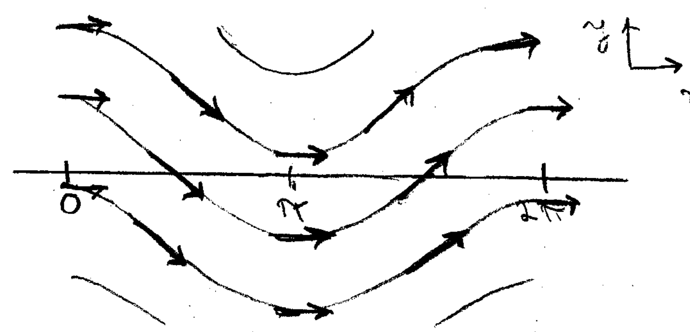


At $p = 500\,\text{hPa}$ we have

$$\Phi = \Phi_0\big|_{500} - \frac{U}{2} f_0 y + \frac{U}{2k} f_0 \sin\left(kx + \frac{\pi}{2}\right)$$

$$u_g = \frac{U}{2}$$

$$v_g = \frac{U}{2}\cos\left(kx + \frac{\pi}{2}\right)$$



Note that to sketch Φ contours in this case we set

$$\Phi = \Phi\big|_{500} - \frac{U}{2} f_0 y + \frac{U}{2k} f_0 \sin\left(kx + \frac{\pi}{2}\right) = \text{Const}$$

implying

$$y = \frac{1}{k}\sin\left(kx + \frac{\pi}{2}\right) + \text{Const}$$

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b

To get T we take

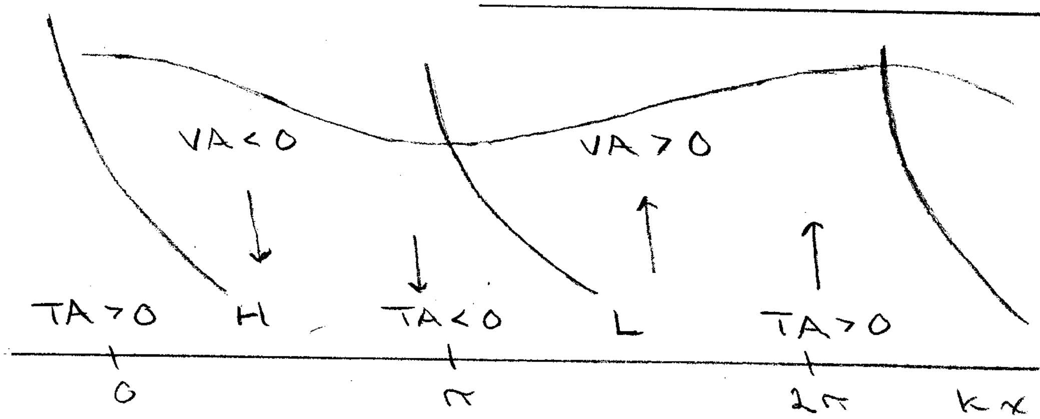
$$T = -\frac{p}{R}\frac{\partial \Phi}{\partial p} = -\frac{p}{R}\left[\frac{d\Phi_0}{dp} + U f_0 y \frac{\pi}{3}\frac{1}{p_0}\sin\left(\frac{\pi}{3}\frac{p}{p_0} + \frac{\pi}{6}\right)\right.$$
$$\left. - \frac{U}{2k} f_0 \pi \frac{1}{p_0}\cos\left(kx + \pi\frac{p_0 - p}{p_0}\right)\right]$$

which at $p = p_0$ gives

$$T\Big|_{p_0} = -\frac{p_0}{R}\frac{d\Phi_0}{dp} - \frac{U f_0 y}{R}\frac{\pi}{3} + \frac{U}{2k}\frac{f_0}{R}\pi\cos(kx)$$

Our TA must then be (at $p = p_0$)

$$TA = -u_g\frac{\partial T}{\partial x} - v_g\frac{\partial T}{\partial y} = -v_g\frac{\partial T}{\partial y}$$
$$= \frac{U}{2}\cos(kx)\,\frac{U f_0}{R}\frac{\pi}{3} = \frac{U^2}{2}\frac{f_0}{R}\frac{\pi}{3}\cos(kx)$$



c

For the vorticity we have

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{\partial v_g}{\partial x} = -\frac{U}{2}k\sin\left(kx + \pi\frac{p_0 - p}{p_0}\right)$$

s.t. at $p = p_0/2$

$$VA = -u_g\frac{\partial \zeta_g}{\partial x} - v_g\frac{\partial \zeta_g}{\partial y} = -u_g\frac{\partial \zeta_g}{\partial x}$$
$$= \frac{U^2}{4}k^2\cos\left(kx + \frac{\pi}{2}\right)$$

d

In qualitative form, our tendency eqn looks like

$$\frac{\partial \Phi}{\partial t} \sim -VA + \frac{\partial}{\partial p} TA$$

s.t.

i) The TA at the surf is in phase with the upper-level ridges and troughs. The warm advection below the ridges will make Φ in the ridge higher ($\partial \Phi/\partial t > 0$) while the cold advection below the trough will make the trough deeper ($\partial \Phi / \partial t < 0$). The TA thus acts to amplify the wave.

ii) The 500 hPa VA is out of phase with the ridges and troughs. The positive VA ahead of the trough will tend to shift the trough eastward ($\partial \Phi/\partial t < 0$) while the negative VA ahead of the ridge will tend to shift the ridge eastward. The VA thus acts to propagate the wave.

e

The ω-eqn in qualitate form looks like

$$\omega \sim -\frac{\partial}{\partial p} VA + TA.$$

So i) positive VA above you implies upward motion and ii) positive TA implies upward motion (as sketched).