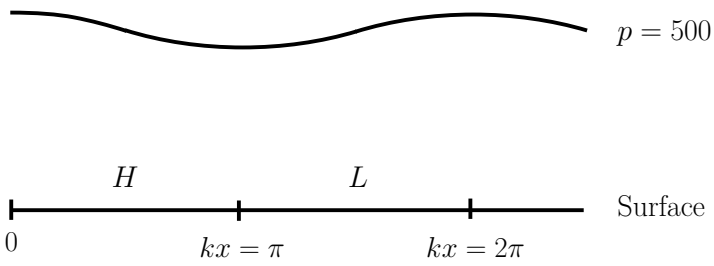
Atmospheric Sciences 435, Spring 2008
Problem Set 8
Due Tuesday, Apr 29

Problem 1 *Diagnosis of a simple wave structure*
The figure below shows a schematic cross-section through a developing baroclinic disturbance. The centers of high and low pressure at the ground are denoted by H and L, while the wavy solid line suggests the geometry of the 500 hPa pressure surface. At the instant shown in the figure the geopotential associated with the disturbance is

$$\Phi(x,y,p) = \Phi_0(p) - U f_0\, y\, \cos\left(\frac{\pi}{3}\frac{p}{p_0} + \frac{\pi}{6}\right) + \frac{U}{2k} f_0\, \sin\left(kx + \pi\,\frac{p_0 - p}{p_0}\right)$$

where $p_0 = 1000$ hPa and where f_0 and U are both positive constants. The function $\Phi_0(p)$ is the background geopotential associated with the rest state.



Assuming hydrostatic and quasi-geostrophic dynamics, derive the following properties of the flow field:

(a) Find the geostrophic wind (u_g, v_g). Qualitatively sketch (i) the zonal wind profile $u_g(p)$ for 1000 hPa $\geq p \geq$ 250 hPa; and (ii) the vector (u_g, v_g) and contours of constant geopotential Φ on both the 1000 hPa and 500 hPa pressure surfaces (i.e., as a function of x and y).

(b) Find the temperature advection $-u_g\,\partial T/\partial x - v_g\,\partial T/\partial y$ as evaluated at the surface (i.e., for $p = p_0$). Reproduce the vertical cross section given above and indicate on your sketch the regions of maximum (most positive) and minimum (most negative) temperature advection (TA) at the surface.

(c) Find the geostrophic vorticity advection $-u_g\,\partial\zeta_g/\partial x - v_g\,\partial\zeta_g/\partial y$ as evaluated at 500 hPa. Indicate the regions of maximum (most positive) and minimum (most negative) vorticity advection (VA) on your sketch.

(d) State the geopotential tendency equation in its qualitative form. (i) Based on the tendency equation, how should the TA at the surface effect the ridges and troughs at 500 hPa? Are the TA and the ridge / trough patterns in phase or out of phase? (ii) How will the 500 hPa VA effect the ridges and troughs? (You can ignore the β term here.) Are the VA and ridge / trough patterns in phase or out of phase? Explain your answers.

(e) State the qualitative form of the omega equation. Based on your answers to parts (b) and (c), where on your sketch do you expect upward and downward motion? Indicate with vertical arrows.