

Atmospheric Sciences 435, Spring 2008
Problem Set 6
Due Monday, Apr 14

Problem 1: *The updraft-shear feedback*

Recall that the dynamic part of the pressure P_D is determined by the dynamic pressure equation

$$\nabla^2 P_D = -\frac{|def|^2}{2} + \frac{|\boldsymbol{\omega}|^2}{2} \quad (1)$$

where

$$|def| = \left[2 \left(\frac{\partial u}{\partial x} \right)^2 + 2 \left(\frac{\partial v}{\partial y} \right)^2 + 2 \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \right]^{1/2}$$

is a measure of deformation in 3D, while

$$|\boldsymbol{\omega}| = \left[\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right)^2 + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)^2 \right]^{1/2}$$

is the length of the vorticity vector.

(a) Consider an east-west background shear profile as given by

$$\mathbf{U}(z) = U(z) \hat{\mathbf{x}} = \Lambda z \hat{\mathbf{x}}$$

where Λ is a positive constant and where $\hat{\mathbf{x}} = (1, 0, 0)$ is the standard unit vector in the x direction. Show that the vorticity and deformation terms for this shear profile cancel exactly so that (1) reduces to $\nabla^2 P_D = 0$.

(b) Now consider an axisymmetric updraft of the form

$$\mathbf{u} = w(x, y) \hat{\mathbf{z}} = w_0 e^{-x^2-y^2} \hat{\mathbf{z}}$$

where w_0 is again a positive constant. Show that for this flow field (without the background shear profile) the deformation and vorticity terms again cancel so that (1) again reduces to $\nabla^2 P_D = 0$.

(c) So far neither the background shear profile nor the updraft have by themselves produced an anomaly in pressure. But now suppose we superpose the two—that is, suppose the net flow field is given by

$$\mathbf{u} = \Lambda z \hat{\mathbf{x}} + w_0 e^{-x^2-y^2} \hat{\mathbf{z}} \quad (2)$$

Compute $|def|^2$ and $|\boldsymbol{\omega}|^2$ and show that on the east side of the the updraft (i.e., for $x > 0$) we have $|\boldsymbol{\omega}|^2 > |def|^2$, while on the west side ($x < 0$) we have $|def|^2 > |\boldsymbol{\omega}|^2$. That is, the vorticity is enhanced on the downshear side of the updraft while the deformation is enhanced on the upshear side. (Refer to pg. 15 of the storm structure notes.)

(d) Given the basic rule of thumb that anomalies in P_D tend to be opposite in sign to $\nabla^2 P_D$, do you expect the pressure on the east side of the updraft to be increased or decreased by the updraft-shear feedback? On the west side?

(e) Now suppose we kick things up a notch by letting the background shear profile curve with height (as found in the climatological profiles). Specifically, consider

$$\mathbf{U}(z) = U(z) \hat{\mathbf{x}} + V(z) \hat{\mathbf{y}} = \begin{cases} -U_0 \cos(\frac{\pi z}{H}) \hat{\mathbf{x}} + U_0 \sin(\frac{\pi z}{H}) \hat{\mathbf{y}} & \text{for } z \leq H \\ U_0 \hat{\mathbf{x}} & \text{otherwise} \end{cases} \quad (3)$$

where U_0 and H are both constants. (Refer to pg. 16 of the storm structure notes.) Sketch the wind profile as a function of z by plotting the wind vector (3) at $z/H = 0, 1/6, 1/3, 1/2, 2/3, 5/6$ and 1.

(f) Adding in the updraft as in (2), compute $|def|^2$ and $|\boldsymbol{\omega}|^2$ at $z/H = 0$ and show that on the north side of the updraft ($y > 0$) we have $|\boldsymbol{\omega}|^2 > |def|^2$ while on the south side $|def|^2 > |\boldsymbol{\omega}|^2$. On which side of the updraft will the pressure be decreased? On which side increased?

(g) Now compute $|def|^2$ and $|\boldsymbol{\omega}|^2$ at $z/H = 1$ [assume z/H is actually very slightly less than 1, so that only the first case in (3) actually matters] and show that the relations in this case are reversed: that is, on the north side of the updraft $|def|^2 > |\boldsymbol{\omega}|^2$ while on the south side $|\boldsymbol{\omega}|^2 > |def|^2$. On which side is the pressure decreased at this height? Increased?

(h) Make a sketch in the xy plane showing the background velocity vectors \mathbf{U} , the location of the updraft maximum and the regions of increased (indicate by H) and decreased (L) pressure anomalies for $z/H = 0$. Repeat for $z/H = 1$.

(i) Consider the vertical pressure gradients produced by the pressure anomalies sketched in (h). Is the vertical PGF on the south side of the updraft positive (upward) or negative (downward)? On the north side?

(j) Broadly speaking, the splitting of a supercell storm is determined primarily by vortex stretching and tilting (see pg. 12 of the storm structure notes), with the updraft-shear interaction described above providing a secondary feedback. Based on your answer to (i), would you expect this updraft-shear feedback to favor the north side of the split storm or the south side? Explain your answer.