

Atmospheric Sciences 435, Spring 2008
Problem Set 4
Due Monday, Mar 24

Problem 1: *Wave dispersion in deep water*

Previously we considered sinusoidal surface waves where the wavelength of the waves was much greater than the fluid depth (i.e., the *shallow-water* case). Shallow-water flows provide a realistic description for many long-wavelength ocean waves—such as those produced by tidal forces or earthquakes—and are also a reasonable analog for many gravity-driven atmospheric motions. However, the ocean waves we typically see by eye (when out fishing, say) generally have much shorter wavelengths. These shorter waves differ from the shallow-water case in two key respects: (i) since the fluid depth is not small compared to the wavelength, the hydrostatic approximation does not apply; and (ii) the waves typically oscillate much faster than once-per-day and the effects of Coriolis are thus small.

(a) It turns out that if we neglect Coriolis and do not make the hydrostatic assumption, then surface gravity waves satisfy the general dispersion relation

$$\omega^2 = gk \frac{e^{kH} - e^{-kH}}{e^{kH} + e^{-kH}} \quad (1)$$

where H is the resting depth of the fluid. As a check, show that (1) reduces to the shallow-water dispersion relation (with $f = 0$) in the limit where the wavelength is much larger than the fluid depth. (Remember your Taylor series!)

(b) Show that for the opposite limit (i.e., for short waves with $\lambda \ll H$) the dispersion relation in (a) is approximated by

$$\omega^2 = gk \quad (2)$$

(*Hint:* just consider the $H/\lambda \rightarrow \infty$ limit.)

(c) The waves we see when out fishing typically have wavelengths similar to or somewhat smaller than the fluid depth H . Clearly the approximation in (b) holds for the shorter waves, but how about for $\lambda \sim H$? Find the fractional error in (2) [relative to (1)] for the case $\lambda = H$ and thus show that the approximation in (b) is in fact satisfactory.

(d) The typical wavelength of wind-driven ocean waves turns out to be roughly 100 m, while a typical water depth (on the continental shelf, at least) is also about 100 m. Such waves are thus well approximated by (2). What is a typical period for these waves?

(e) Are the waves described by (2) dispersive? Why or why not?

(f) Suppose that a passing ship creates the wave pattern shown below. Do you expect this



pattern to maintain its shape as the disturbance propagates? If not, which do you expect to move faster: the individual wave crests or the general disturbance envelope?