Atmospheric Sciences 435, Spring 2008
Problem Set 3
Due Monday, Feb 25

Problem 1 *Which is faster: a tsunami or a commercial jet?*

In class we showed that if we include rotation then the freqency and wavelength of a shallow water gravity wave are related by

$$\omega^2 = gHk^2 + f^2$$

where f is the constant Coriolis parameter. Suppose that we apply this to the case of a long ($\lambda \gg H$) tsunami wave propagating on the open ocean. We'll take the depth of the ocean to be roughly 5 km and let the latitude be 45°N.

(a) What is the distance (in km) from New York to London? (*Hint*: Google does nicely here.)

(b) For waves on the deep ocean we can typically ignore the term involving the Coriolis parameter. To see this, set the wavelength equal to the distance described in (a) and then compare the sizes of the gHk^2 and f^2 terms. Would waves with shorter wavelengths then have larger or smaller gHk^2?

(c) Show that if we ignore the Coriolis term then the phase speed of the wave is independent of its wavelength. What is this phase speed for the parameters given above? How long would it take for the wave to cross the Atlantic?

(d) Which is faster: a tsunami wave or a commercial jet?

Problem 2 *Potential vorticity: an exclusive first look*

The shallow-water approximation as derived in class is given by

$$\frac{Du}{Dt} = -g\frac{\partial h}{\partial x} + fv \quad , \quad \frac{Dv}{Dt} = -g\frac{\partial h}{\partial y} - fu \tag{1}$$

$$\frac{Dh}{Dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{2}$$

Recall that the disturbance is independent of z so that the material derivative takes the form

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$$

(a) Show that the momentum equations in (1) can be combined to give the vorticity equation

$$\frac{D}{Dt}(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{3}$$

where $\zeta = \partial v/\partial x - \partial u/\partial y$ is the relative vertical vorticity. (*Hint*: keep in mind that the Coriolis parameter varies with latitude so that $f = f(y)$. And make sure to use the product rule on all the product terms.)

(b) Show that the vorticity equation (3) can then be combined with the height equation (2) to give

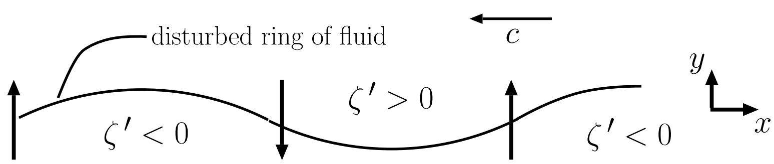
$$\frac{D}{Dt}\left(\frac{\zeta + f}{h}\right) = 0 \tag{4}$$

[*Hint*: recall that for any function $\phi(x, y)$ we have $(1/\phi)D\phi/Dt = D(\ln\phi)/Dt$.] The quantity $(\zeta + f)/h$ is referred to as the *potential vorticity*, and (4) states that this potential vorticity quantity remains constant following fluid particles.

(c) Consider a fluid column with local convergence so that h is increasing with time (i.e., $Dh/Dt > 0$). Will the absolute vorticity $\zeta + f$ for this column get larger or smaller?

Problem 3 *Rossby waves in the Southern Hemisphere*

The figure below shows the phase relationships for a shallow-water Rossby wave in the Northern Hemisphere. As shown in class, Rossby waves in the Northern Hemisphere always propagate westward relative to the mean current. Which direction do Rossby waves propagate in the Southern Hemisphere? Explain your answer.



Problem 4 *The long and short of it all*

In class we showed that the dispersion relation for a 2D shallow-water Rossby wave propagating on a uniform background current U is given by

$$\omega = Uk - \frac{\beta}{k}$$

To be concrete, we'll let the current speed be given by $U = 10$ m/s and let the disturbance be centered at $\phi = 60^{o}$N so that $\beta \approx 1.14 \times 10^{-11}$.

(a) Show that for short wavelengths the phase speed of the wave (measured relative to the ground) is essentially the current speed U, but that as the wavelength increases the phase speed becomes smaller than U.

(b) For what wavelength is the phase speed zero? Do longer waves propagate eastward or westward?

(c) Compare the phase speeds of waves with wavelengths of 1000 km, 4000 km and 10000 km.