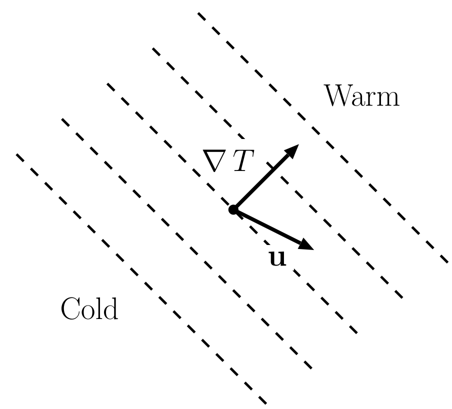
Atmospheric Sciences 435, Spring 2008
Problem Set 1
Due Friday, Feb 1

Problem 1 *Blowing in the wind.....*



Suppose the temperature is increasing toward the northeast at 0.05 K/km and that the wind is directed toward the east-southeast (i.e., at an angle of 22.5° to the x-axis) at 8 m/s (see illustration at right).

(a) As an approximation, suppose we assume that the temperature is not changing following fluid particles; that is, suppose that

$$\frac{DT}{Dt} = 0$$

Then what is the rate of temperature change as observed at a fixed point?

(b) Now suppose that due to solar radiation the temperature following fluid particles is increasing at 1 K/hr; that is, suppose that

$$\frac{DT}{Dt} = Q$$

where Q is 1 K/hr. Then what is the rate of temperature change (again observed at a fixed point) in this case?

Problem 2 *But then how would I talk?*

We argued in class that to remove the sound waves from the equations of motion we should replace the full pressure-divergence equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{1}{\rho c_s^2}\frac{Dp}{Dt}$$

with the anelastic version

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - \frac{g}{c_s^2} w = 0$$

where $c_s^2 = (c_p/c_v)RT$ is the speed of sound squared. Here we explore this approximation a bit further in terms of the expansion and compression of fluid particles.

(a) In 336 we showed that for 2D flow the divergence is related to the area of a fluid particle by

$$\frac{1}{A}\frac{DA}{Dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

where $A = \Delta x \Delta y$ is the area. In the 3D case the equivalent relation is

$$\frac{1}{V}\frac{DV}{Dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

where $V = \Delta x \Delta y \Delta z$ is now the particle volume. With this in mind, show that the anelastic approximation predicts that ascending fluid particles must expand. Does this seem reasonable (i.e., does that really happen)?

(b) Consider an ascending fluid particle starting with volume V_0 at height z_0 and then ascending to height z where the volume is $V(z)$. Assuming that c_s^2 is roughly constant during the ascent, show that the anelastic approximation predicts

$$V(z) = V_0 \, e^{\,g(z-z_0)/c_s^2}$$

(*Hint:* remember that $(1/V)DV/Dt = D(\ln V)/Dt$.)

(c) Suppose that the temperature of the ascending fluid particle stays roughly constant at $T = 273$ K (an approximation, of course). By what fraction does the volume of the fluid particle increase [i.e., what is $(V(z) - V_0)/V_0$] for an ascent of 100 m? For an ascent of 1 km? And 10 km?

(d) Now suppose we make the incompressibility approximation instead. By what fraction does the volume of the ascending fluid particle increase in this case?