1

Given the 500 hPa geopotential distribution

$$\Phi(x,y) = \Phi_r - f U y + f \frac{U}{2k} \sin(kx)\cos(ly)$$

with

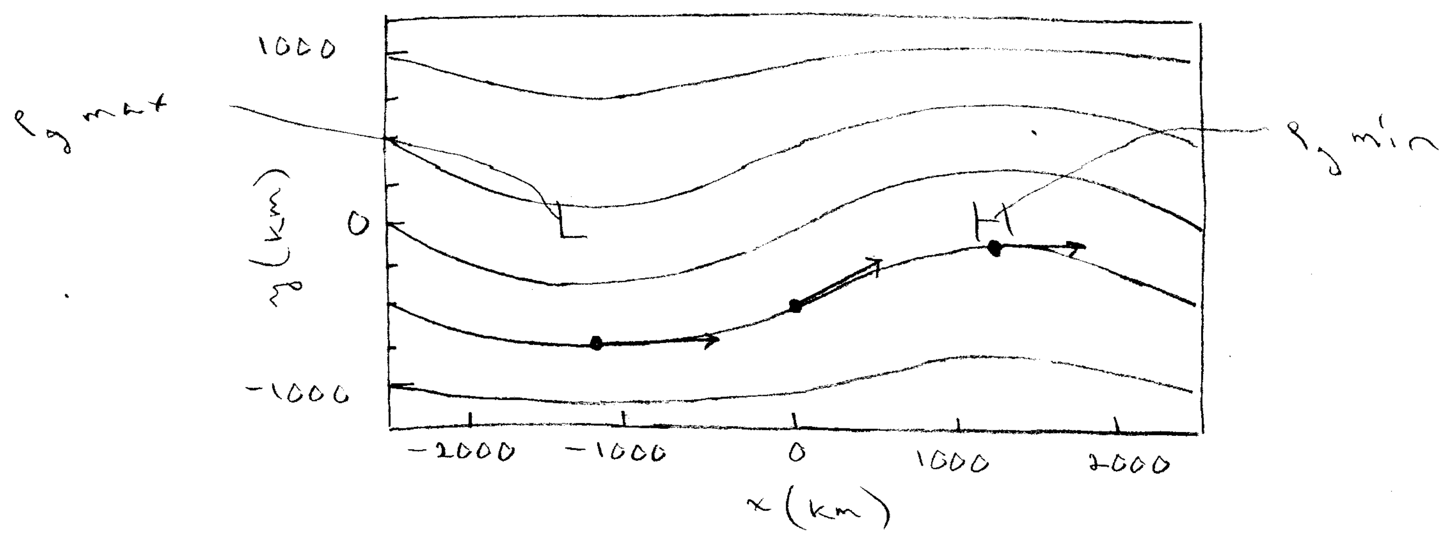
$$\Phi_r = g \times (5\,\text{km}), \quad f = 10^{-4}\,\text{s}^{-1}, \quad U = 20\,\text{m/s}$$

$$k = l = 2\pi/5000\ \text{km}^{-1}$$

a

The disturbance geopotential is defined by

$$\Phi' = \Phi - (\Phi_r - f U y) = f \frac{U}{2k} \sin(kx)\cos(ly)$$



The max then occurs where $\sin(kx) = \cos(ly) = 1$ while the min occurs where $\cos(ly) = 1$ and $\sin(kx) = -1$.

b

The geostrophic balance in pressure coordinates implies

$$u_g = -\frac{1}{f}\frac{\partial \Phi}{\partial y} = -\frac{1}{f}\left(-fU - f\frac{U}{2}\sin(kx)\sin(ly)\right)$$

$$= U + \frac{U}{2}\sin(kx)\sin(ly)$$

$$v_g = \frac{1}{f}\frac{\partial \Phi}{\partial x} = \frac{1}{f}\left(f\frac{U}{2}\cos(kx)\cos(ly)\right)$$

$$= \frac{U}{2}\cos(kx)\cos(ly)$$

b [cont]

That is

$$\underline{V}_g = (u_g, v_g)$$
$$= U\hat{x} + \frac{U}{2}\left(\sin(kx)\sin(ly),\ \cos(kx)\cos(ly)\right)$$

- A: $(-1250, -750)$ or $kx \simeq -\frac{\pi}{2}$, $ly \simeq -\frac{\pi}{3.3}$

$$(u_g, v_g) \simeq U\hat{x} + \frac{U}{2}(0.8, 0) = 1.4U\hat{x}$$
$$|\underline{V}_g| = 1.4U$$

- B: $(0, -500)$ or $kx = 0$, $ly \simeq -\frac{\pi}{5}$

$$(u_g, v_g) \simeq U\hat{x} + \frac{U}{2}(0, 0.8) = (U, 0.4U)$$
$$|\underline{V}_g| = (U^2 + 0.16U^2)^{1/2} \simeq 1.08U$$

- C: $(1250, -250)$ or $kx \simeq \frac{\pi}{2}$, $ly \simeq -\frac{\pi}{10}$

$$(u_g, v_g) \simeq U\hat{x} + \frac{U}{2}(-0.3, 0) = 0.85U\hat{x}$$
$$|\underline{V}_g| = 0.85U$$

For the sketch we have $\underline{V}_g$ parallel to contours of constant Φ with

$$|\underline{V}_g|_A > |\underline{V}_g|_B > |\underline{V}_g|_C$$

c

Here

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$
$$= -\frac{Uk}{2}\sin(kx)\cos(ly) - \frac{Ul}{2}\sin(kx)\cos(ly)$$

or since $l = k$

$$\zeta_g = -Uk\sin(kx)\cos(ly) = -\frac{2k^2}{6}\Phi'$$

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c [cont]

The vorticity is thus max where Φ' is min and vice-versa.

1

We'll adopt pressure coords from the start, s.t.

$$\Phi = \Phi(x, y, p, t), \quad T = T(x, y, p, t), \quad \text{etc}$$

and we'll adopt the notation

$$\underline{\frac{\partial \Phi}{\partial y} : \text{horizontal derivative at fixed } p} = \left(\frac{\partial \Phi}{\partial y}\right)_p$$

and similarly for the other variables. Then by the rules of calculus

$$\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial \Phi}{\partial p}\right) \quad \text{etc}$$

a

Our geostrophic and hydrostatic balance conditions in pressure coords are

$$\underbrace{f u_g = -\frac{\partial \Phi}{\partial y}, \quad f v_g = \frac{\partial \Phi}{\partial x}}_{\text{geostrophic}}, \quad \underbrace{\frac{\partial \Phi}{\partial p} = -\frac{1}{\rho} = -\frac{RT}{p}}_{\text{hydrostatic}}$$

We must then have

$$\frac{\partial u_g}{\partial p} = -\frac{1}{f}\frac{\partial}{\partial p}\left(\frac{\partial \Phi}{\partial y}\right) = -\frac{1}{f}\frac{\partial}{\partial y}\left(-\frac{RT}{p}\right)$$

But since $\partial/\partial y$ is at fixed pressure, we can pull the p out of the derivative and write

$$\underline{\frac{\partial u_g}{\partial p} = \frac{R}{fp}\frac{\partial T}{\partial y}}$$

and a similar argument for v_g shows

$$\underline{\frac{\partial v_g}{\partial p} = \frac{-R}{fp}\frac{\partial T}{\partial x}}$$

Now to convert the p derivative to a z derivative we use

$$\frac{\partial u_g}{\partial z} = \frac{\partial u_g}{\partial p}\frac{\partial p}{\partial z} = -\rho g \frac{\partial u_g}{\partial p}$$

a [cont]

which implies

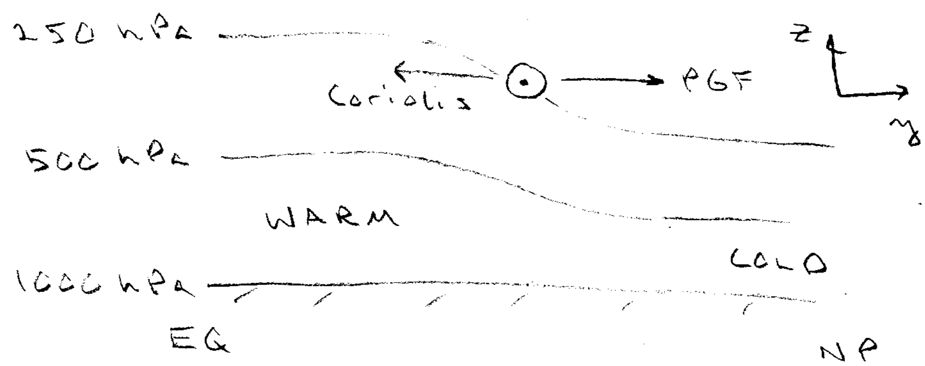
$$\frac{\partial u_g}{\partial z} = -\rho g \frac{R}{f p} \frac{\partial T}{\partial y} = -\frac{g}{f T} \frac{\partial T}{\partial y}$$

$$\frac{\partial v_g}{\partial z} = -\rho g \left(-\frac{R}{f p}\right) \frac{\partial T}{\partial x} = \frac{g}{f T} \frac{\partial T}{\partial x}$$

showing that the vertical shear of the geostrophic wind is determined completely by the horizontal temperature gradient at fixed pressure

b

By assumption, our lowest pressure surface coincides roughly with the ground s.t. Φ on this surface is roughly constant. That is



$$u_g\Big|_{p=p_{srf}} = -\frac{1}{f}\frac{\partial \Phi}{\partial y}\Big|_{p=p_{srf}} \simeq 0$$

where p_{srf} is the surface pressure. But also by assumption we know

$$\frac{\partial T}{\partial y} < 0 \quad \text{s.t.} \quad \frac{\partial u_g}{\partial z} = -\frac{g}{f T}\frac{\partial T}{\partial y} > 0$$

So all together our winds are weak near the surface but become more westerly (positive u_g) with height.