Computing the Jet Stream
(Atmo 336, Fall 2007)

Set Up: In two previous labs we introduced methods for approximating an integral as a sum (the trapezoidal rule) and several methods for approximating a spatial derivative using finite differences (the centered difference, the one-sided difference, etc.). Here we combine these methods to compute the pressure and wind distributions for an idealized atmospheric cross-section.

Getting Started: The basic premise is this: suppose we have a set of high-resolution temperature observations from the ATMO336 satellite and we wish to approximate the associated wind fields to get an estimate of the jet stream height. The satellite-derived data is for a meridional (yz) cross-section at fixed x and is stored in the MATLAB mat file *T.mat* in the shared ATMO336 directory (under *lab_7_stuff*). (True to our roots as dynamicists, the 'data' in *T.mat* was actually computed from an analytic formula. But for our purposes we'll treat the data as real.) The data covers the region $0 \le z \le H$ and $-L/2 \le y \le L/2$ and is stored on a grid consisting of $2ny + 1$ points in the horizontal and $nz + 1$ points in the vertical. The associated y and z coordinates for the grid are stored in the mat files *y.mat* and *z.mat*, also in the shared directory.

Copy the temperature file and the grid position files to your home directory and then take a look:

```
load T;
load y;
load z;
v = 135:10:305;
[CT hT] = contour(y,z,T,v);
set(hT,‘linewidth’,2);
clabel(CT,hT,‘labelspacing’,450);
```

The first three commands read in data matrices of size $(2ny + 1) \times (nz + 1)$ from the corresponding mat files and the remaining commands then make the contour plot.

To compute the wind field from the temperature we'll make use of our large-scale balance approximations. The computation proceeds in two steps: (i) first we integrate the temperature vertically assuming hydrostatic balance to get the pressure—and by extension density—fields; and then (ii) we compute the horizontal pressure-gradient force (PGF) using finite differences and recover the winds by assuming geostrophic balance.

First an Integral: Ok, first the integral. Recall that the hydrostatic balance relation can be written in the form

$$\frac{\partial \ln p}{\partial z} = -\frac{g}{RT} \tag{1}$$

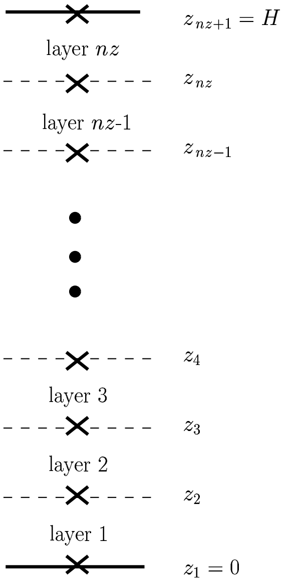
where here T is a function of y and z. When we integrated this relation previously (in the hypsometric lab, I believe) our objective was to find the height of a specified pressure surface given the distribution of T. But now our objective is the opposite—that is, now we want the pressure as a function of z rather than z as a function of p. But the basic approach is the same.

(a) Start by showing that the integral of (1) leaves us with

$$p(y,z) = p_s(y)\, e^{-I(y,z)} \qquad \text{where} \qquad I(y,z) = \int_0^z \frac{g}{R\,T(y,z')}\, dz' \tag{2}$$

and where $p_s(y) = p(y, z=0)$ is the surface pressure. We'll have to assume something for the y-dependence of p_s, so to keep things simple we'll assume a constant value $p_s = 1000$ hPa (which is 10^5 Pa, of course).

Next we divide each column of our yz cross-section into nz layers (corresponding to the $nz+1$ data points in the vertical) as illustrated below. To get the value of p at the level n we need to integrate the exponent in (2) between 0 and the height $z_n = (n-1)\,dz$. As before we use the trapezoidal rule, which involves approximating the integrand for each layer using the average of the values at the top and bottom of the layer. Using this approximation for the integral in (2) then leads us to



$$p(z_n) \approx p_s e^{-I_n} \qquad \text{where} \qquad I_n = \frac{g}{R} \sum_{k=1}^{n-1} \frac{2}{T(z_{k+1}) + T(z_k)}\, \Delta z \tag{3}$$

where Δz is the distance between the grid points and where we have suppressed the y-dependence with the understanding that we integrate upward at fixed y.

Let's write a short program (*pressure.m*, say) to get the pressure distribution as a function of y and z. For notation, let $T(j,n)$ represent the value of T at the j^{th} gridpoint in y and the n^{th} gridpoint in z—that is, at point $y = -L/2 + (j-1)\,\Delta y$ and $z = (n-1)\,\Delta z$. Then to get the pressure for all y and z we'll use the nested loop

```
load T;
load y;
load z;

ny = 50;
nz = 50;
ps = 100000.;
R = 287.06;
g = 9.806;
H = 18000.;

I = zeros(2*ny+1,nz+1);
p = zeros(2*ny+1,nz+1);

dz =      ;

for j=1:2*ny+1
  I(j,1) = 0;
  for n=1:nz
    I(j,n+1) = I(j,n) +      ;
  end
end

p = ps*exp(-I);
```

where of course you'll need to fill in the blanks for both the integral and for dz.

(b) Plot your temperature and pressure fields by copying the script *plot_Tp.m* to your home directory and running *plot_Tp* on the command line. The dotted field shows the temperature distribution and the solid shows the pressure. Save your plot to *Temp_press.jpg* and copy to the figure directory to turn in. And then answer the following:

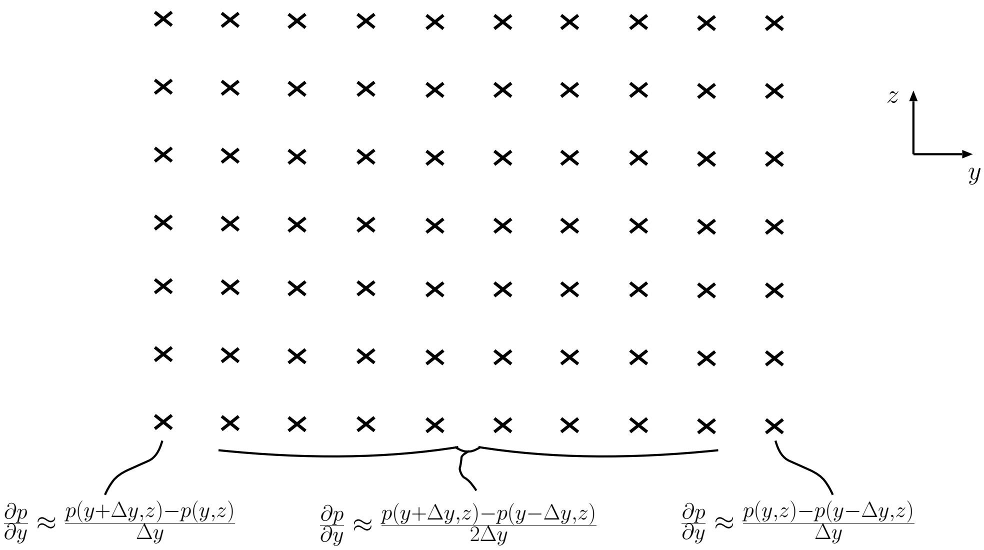
- Where are the pressure contours furthest apart? Closest together?
- How is the spacing of the pressure contours related to the temperature field?
- The height of the jet stream is typically somewhere around 10 to 12 km. Is the pressure at this height higher on the equator side or the pole side?
- Which direction is the horizontal PGF at this height—equatorward or poleward?

And Then a Derivative: Ok, we have both the pressure and temperature fields. To get the wind we now need to compute the pressure gradient force and then approximate u using geostrophic balance; i.e.,

$$fu \approx -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \text{or} \qquad u \approx -\frac{1}{f\rho}\frac{\partial p}{\partial y}$$

To keep things simple we'll assume that f is a constant with value of $f = 10^{-4}$ s^{-1}. Then to get the PGF we first compute the density and then approximate the pressure derivative as a finite difference.

In a previous lab we showed that our centered difference approximation for the derivative is more accurate than a one-sided approximation, so we'll compute our pressure derivative using centered differences. But we have to be a bit careful, because a centered approximation requires grid points to both the left and right of the grid point of interest, whereas at the edges of our grid we only have grid points to one side or the other. So to get around this inconvenience we'll use a centered difference to compute the derivative everywhere on the interior of the grid but we'll use one-side approximations at the north and south grid boundaries. Our complete method for computing the derivative is then as illustrated below.



$$\frac{\partial p}{\partial y} \approx \frac{p(y+\Delta y,z)-p(y,z)}{\Delta y} \qquad \frac{\partial p}{\partial y} \approx \frac{p(y+\Delta y,z)-p(y-\Delta y,z)}{2\Delta y} \qquad \frac{\partial p}{\partial y} \approx \frac{p(y,z)-p(y-\Delta y,z)}{\Delta y}$$

To implement our computation we do something like the following (assuming you've already run your *pressure.m* script), which you should probably save as *wind.m*:

```
f = 1./10000.;
L = 4.*10^6;
dy = L/(2*ny);

rho = zeros(2*ny+1,nz+1);
dpdy = zeros(2*ny+1,nz+1);
```

```
% south edge
j = 1;
for n=1:nz+1
   dpdy(j,n) =      ;
   rho(j,n) =      ;
   u(j,n) =     ;
end

% north edge
j = 2*ny+1;
for n=1:nz+1
   dpdy(j,n) =      ;
   rho(j,n) =      ;
   u(j,n) =     ;
end

% interior
for j=2:2*ny
for n=1:nz+1
   dpdy(j,n) =      ;
   rho(j,n) =      ;
   u(j,n) =     ;
end
end
```

where of coure you will again have to fill in the blanks.

(c) Now to plot the fruits of our labor. Copy the *plot_wind.m* script to your home directory and run *plot_wind* on the command line. The dotted/color-filled contours are again the temperature, while this time the red lines show the pressure while the black lines show the wind. Save your plot to *jet.jpg* and copy to the figure directory to turn in. And then answer the following:

- Which direction is the wind blowing: into or out of the screen?
- At what height is the wind strongest? And how is this height related to the slope of the pressure surfaces?
- At what position in y is the wind strongest? And how is this related to the near-surface temperature gradient?
- Observations show that global warming tends to produce larger temperature increases at the poles than at the equator. In the context of our simple calculation, would you expect this to increase or decrease the strength of the jet? Can you think of a way to show this by modifying the data in the lab?