

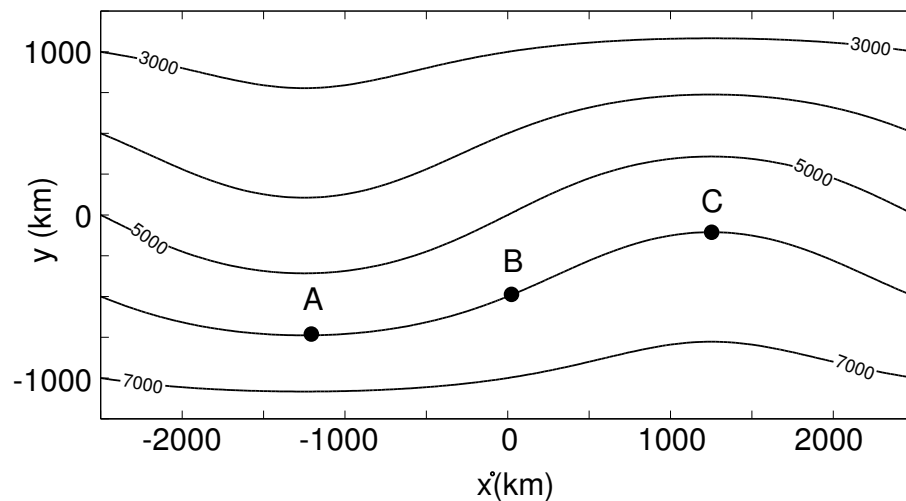
Atmospheric Sciences 336, Fall 2007
Problem Set 7
Due Monday, Dec 3

Problem 1 *Disturbance geopotential and the geostrophic vorticity*

Consider the 500 hPa geopotential distribution defined by

$$\Phi(x, y, p = 500 \text{ hPa}) = \Phi_r - fUy + f \frac{U}{2k} \sin(kx) \cos(ly)$$

where $\Phi_r = g \times (5\text{km})$ is the constant reference geopotential, $f = 10^{-4} \text{ s}^{-1}$ is the Coriolis parameter, $U = 20 \text{ m/s}$ and $k = l = (2\pi/5000) \text{ km}^{-1}$. A contour plot of Φ is given below. (The labels in the contour plot show geopotential height $Z = \Phi / g$ rather than geopotential). For the following, assume that the Coriolis parameter is a constant.



(a) Because of the north-south temperature difference, the 500 hPa pressure surface slopes downward from equator to pole. We might therefore define our background state geopotential to be

$$\Phi_0(y) = \Phi_r - fUy$$

and let the disturbance geopotential be defined by $\Phi' = \Phi - \Phi_0(y)$. Indicate by 'H' and 'L' the regions of highest and lowest disturbance geopotential on the figure. Regions of high Φ' are generally called *ridges* and regions of low Φ' are called *troughs*.

(b) Find the geostrophic wind vector (u_g, v_g) and sketch the wind at the points A, B and C on the figure. (Just try to estimate (x, y) in each case.) Make sure your sketch reflects the relative magnitudes of the wind at the three points.

(c) Find the geostrophic vorticity

$$\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}$$

and indicate by '+' and '-' the regions of maximum and minimum ζ_g on the figure. How are the locations of maximum and minimum ζ_g related to the ridges and troughs?

Problem 2 *The thermal windshear*

In pressure coordinates our vertical coordinate is pressure—that is, we assume $\Phi = \Phi(x, y, p, t)$ (and similarly for other variables). And by the rules of calculus this means that

$$\frac{\partial}{\partial p} \left(\frac{\partial \Phi}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial \Phi}{\partial p} \right)$$

where in both cases the horizontal derivative is implicitly at fixed p .

(a) Using the result given above along with the geostrophic and hydrostatic balance conditions (in pressure coordinates), show that the vertical shear of the geostrophic wind is related to the horizontal temperature gradient along the pressure surface. Specifically, show that

$$\frac{\partial u_g}{\partial z} = -\frac{g}{fT} \left(\frac{\partial T}{\partial y} \right)_p \quad \text{and} \quad \frac{\partial v_g}{\partial z} = \frac{g}{fT} \left(\frac{\partial T}{\partial x} \right)_p$$

where the subscript p indicates a derivative at fixed p . (*Hint*: First take $\partial u_g / \partial p$ and then use

$$\frac{\partial u_g}{\partial z} = \frac{\partial u_g}{\partial p} \frac{\partial p}{\partial z} = -\rho g \frac{\partial u_g}{\partial p}$$

to get $\partial u_g / \partial z$.) This shear / temperature gradient relationship is called the *thermal windshear* equations.

(b) Suppose that the pressure at the ground is roughly uniform with latitude but that the tropospheric temperature decreases from equator to pole. Use the thermal windshear relations derived in (a) to explain why the jet stream blows from the west.