

**Atmospheric Sciences 336, Fall 2007**  
**Problem Set 5**  
**Due Wednesday, Nov 7**

**Problem 1** *The rotating coordinate transformation: derivation*

Consider a vector  $\mathbf{x} = (x, y)$  as measured relative to a coordinate system with stationary axes. Letting  $\theta$  be the angle of this vector relative to the  $x$  axis, the components  $x$  and  $y$  of the vector are given by

$$x = |\mathbf{x}| \cos \theta \quad , \quad y = |\mathbf{x}| \sin \theta$$

where  $|\mathbf{x}|$  is the vector's length.

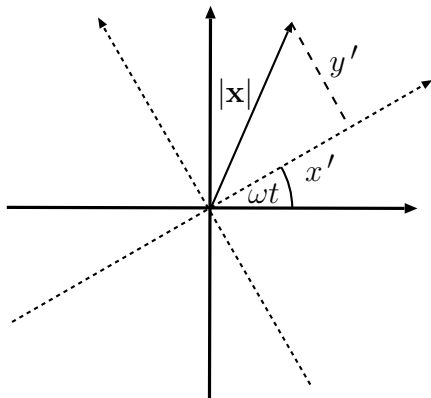
(a) Now consider a new coordinate system (indicated by primes) whose axes rotate at angular velocity  $\omega$  relative to the first system. We suppose that at time  $t = 0$  the two coordinate systems coincide, so that at some later time the angle between the coordinates is  $\omega t$  (as illustrated below) Show that the components of  $\mathbf{x}$  as measured in this new coordinate system are

$$x' = x \cos(\omega t) + y \sin(\omega t) \quad , \quad y' = -x \sin(\omega t) + y \cos(\omega t)$$

where  $x$  and  $y$  are the components as measured in the stationary frame. (*Hint:* remember your trig angle difference formulas; specifically

$$\cos(\beta - \alpha) = \cos \beta \cos \alpha + \sin \beta \sin \alpha \quad \text{and} \quad \sin(\beta - \alpha) = \sin \beta \cos \alpha - \cos \beta \sin \alpha$$

where  $\alpha$  and  $\beta$  are arbitrary angles.)



(b) Now suppose that  $\mathbf{x}$  denotes the position of a fluid particle, so that

$$\mathbf{u} = (u, v) = \left( \frac{dx}{dt}, \frac{dy}{dt} \right)$$

is then the particle's velocity in the stationary frame. Show that the velocity as seen in the rotating frame is given by

$$\begin{aligned} u' &= (u + \omega y) \cos(\omega t) + (v - \omega x) \sin(\omega t) \quad \text{and} \\ v' &= (-u - \omega y) \sin(\omega t) + (v - \omega x) \cos(\omega t) \end{aligned}$$

(c) Take a look at the rotating coordinate lab and make sure the formulas you derived match those used in the lab.

**More fun on the back!**

**Problem 2** *The rotating coordinate transformation: an example*

Consider a particle at rest in the stationary reference frame. Let the position in this frame be given by  $(0, R)$ .

(a) Find the trajectory  $(x'(t), y'(t))$  as viewed from the rotating reference frame and sketch this trajectory over the time interval  $0 \leq \omega t \leq 3\pi/2$ . Does this trajectory match your result from the lab?

(b) At what time does the particle return to its initial position in the rotating frame? Does this make intuitive sense?

(c) Show that the velocity of the particle in the rotating frame is

$$(u', v') = \omega (y', -x')$$

and add velocity vectors to your sketch for the times  $t = 0$ ,  $t = \pi/2$  and  $t = 3\pi/2$ .

(d) Using your results from both (a) and (c), show that the acceleration in the rotating frame can be written as

$$\mathbf{a}' = 2\omega (v', -u') + \omega^2(x', y')$$

Which is bigger in this case: the centrifugal force or the Coriolis force?

**Problem 3** *A high(-to-low) pressure situation*

Consider a fluid particle moving through a pressure distribution described by

$$p(x, y) = -x + \alpha$$

where  $\alpha$  is a constant. At the initial time  $t = 0$  the particle is at rest (i.e.,  $u = v = 0$ ) and located at position  $(x, y) = (-1, 0)$ . Assuming that the only force on the particle is the pressure-gradient force, find the particle's trajectory as a function of time. You may assume that the density of the particle is given by  $\rho = 1$ .