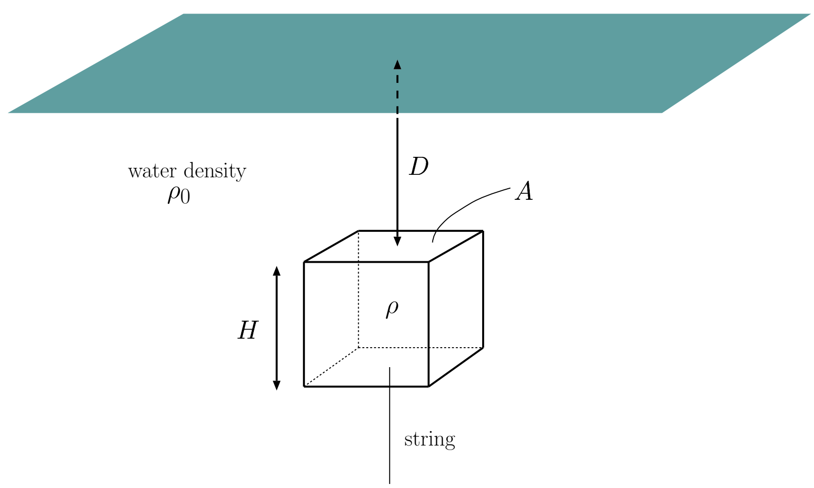
Atmospheric Sciences 336, Fall 2007
Problem Set 2
Due Wednesday, Sept 26

Problem 1 *Riddle me this, Weatherman*
Can an atmosphere in which pressure increases with height be in hydrostatic balance? If so, how?

Problem 2 *Weighing Lake Bryan*
The City of Bryan was asked to report the mass of water in Lake Bryan as part of the biennial Texas water census. But unfortunately the only equipment the city has is an old set of N (where N is large) pressure senors donated by KBTX. Describe an experiment in which the mass of the lake can be determined approximately by placing the N sensors at various points along the lake bottom. (And be careful to explain your instructions clearly—the City of Bryan is counting on you!)

Problem 3 *The block also rises*
Consider a submerged wooden block of density ρ that is tethered to the bottom of a pond by a string (as illustrated below). The block has a height of H and a horizontal cross-sectional area of A. The density of the pond water is ρ_0 (with $\rho_0 > \rho$) and initially the block is located a distance D below the water surface (as measured from the top of the block). At time $t = 0$ the string tether is cut and the block begins to ascend.



(a) First set up the problem. Define a variable to keep track of the position of the block (it's probably simplest to track the block's top) as a function of time. (You can define this variable in any of several ways. Just be clear about what you decide to do.) (i) What is the value of this variable at time $t = 0$? (ii) What is the value of the variable when the block's top first reaches the pond surface?

(b) Now consider the instant just after the tether has been cut but before the block begins to ascend. Define each of the forces acting on the block and show that the net force is given by

$$F^{net} = -\rho' g A H$$

where $\rho' = \rho - \rho_0$ is the density difference between the block and the water. Further show that the acceleration a of the block is given by

$$a = -g\,\frac{\rho'}{\rho}$$

where $-g\rho'/\rho$ is referred to as the block's *buoyancy.*

(c) Strictly speaking, as the block begins to move it will create a pressure disturbance in the water whose net effect is to resist the upward motion. (You can think of this disturbance as a form of drag on the block.) But often the pressure disturbance is small and can to a first approximation be neglected. Suppose that we neglect the pressure disturbance and assume that the pressure is at all times equal to the background hydrostatic pressure in the pond. At what time does the top of the block reach the pond surface?

Problem 4 *The isothermal atmosphere*
Consider a resting atmosphere in which the temperature T is constant with height.

(a) Show that the pressure in this atmosphere decreases exponentially with height. Specifically, show that

$$p(z) = p_s\, e^{-z/H_s}$$

where p_s is the surface pressure and H_s is a constant (called the *scale height*).

(b) An interesting property of exponential decay is that no matter what height z you start at, the vertical distance needed to decrease the pressure by half is the same. (No way!) Verify this is true by finding the heights corresponding to the pressures $p = p_s/2$ and $p = p_s/4$. Express your answers in units of H_s. What would these two heights be for $T = 260$ K?

(c) Latitude-height cross-sections of the atmosphere typically use $\ln p$ as a vertical coordinate rather than p. (Or else they show p in log-coordinates, which is the same thing.) Show that for our isothermal atmosphere changes in $\ln p$ are proportional to changes in height.

Problem 5 *High to low, look out below....*
Consider a resting atmosphere with the temperature profile given by

$$T_0(z) = T_s - \Gamma z$$

where T_s and Γ are constants. Suppose that the pressure at the ground is p_s.

(a) Show that the resting-state pressure distribution is given by

$$p_0(z) = p_s \left(\frac{T_s - \Gamma z}{T_s}\right)^{\frac{g}{R\Gamma}} \tag{1}$$

(*Hint*:

$$\int_{z_1}^{z_2} \frac{1}{a - bz'}\, dz' = -\left.\frac{\ln(a - bz')}{b}\right|_{z_1}^{z_2}$$

where a and b are constants.) If p_s, T_s and Γ are chosen wisely, the resulting temperature and pressure distributions provides a reasonable approximation to the true averaged lower atmosphere. Reasonable 'standard atmosphere' values of these parameters are given by $T_s = 288$ K, $p_s = 1013$ hPa, and $\Gamma = 6.5$ K/km.

(b) For what values of Γ is the resting state described above stable? Is the standard atmosphere value stable?

(c) An altimeter is an instrument that directly measures atmospheric pressure and uses a relation like (1) to estimate the height. [The assumption is that the measured pressure can be taken to be p_0, so that (1) then determines z.] Using the standard atmosphere values of the parameters given above, what altitude does the altimeter read when the pressure is 850 hPa? What about 500 hPa?

(d) Suppose you are flying at constant pressure altitude (as measured by an altimeter like that described above) into an approaching weather system. As you fly, the pressure throughout the lower atmosphere steadily drops due to the change in atmospheric conditions. Is the true height of your plane increasing or decreasing with time?