MORE ABOUT IDEAL GASES

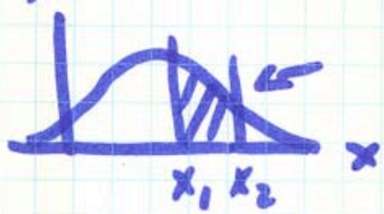
L3 1

EX 2.5 Calc # of collisions/s : 6.5×10^{9} colls/s

2.6 Colls with wall/s : $\approx 10^{27}$

↑ $1\,m^2$

Probability Density Fcns $P(u)$: $\int_{-\infty}^{\infty} P(u)\,du = 1$



$$\int_{x_1}^{x_2} P(x)\,dx = \text{Prob } x \in (x_1, x_2)$$

$\boxed{u = \text{random variable}}$

it takes on diff. values each "realization"

$$\text{mean} = \mu_u = \int_{-\infty}^{\infty} u\,P(u)\,du$$

$$\text{variance} = \sigma_u^2 = \int (u-\mu_u)^2 P(u)\,du$$

ex. uniform pdf

$\mu = \frac{1}{2}$

$\sigma^2 = 1/12 = 0.289$

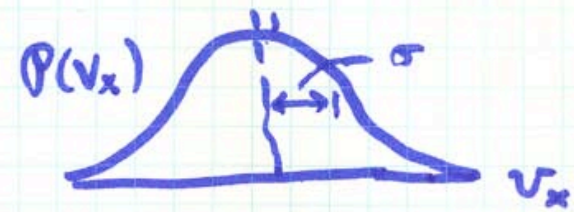
ex. exponential

$P(u) = \frac{1}{b} e^{-u/b}$

$\mu = b$, $\sigma^2 = b^2$

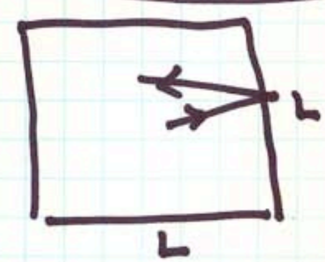
Distribution of v_x

This can be measured: $\quad P(v_x) = \frac{1}{\sqrt{2\pi}\,\sigma_v} e^{-v_x^2/2\sigma^2}$ $\Rightarrow$ Normal Distribution



but what is σ?

Relate pressure to T



Cubic Box side L

v_0 speed of molecule

$$F\,(\text{one molecule}) = \frac{2m_0v_0}{\Delta t}$$

$$\Delta t = \frac{2L}{v_0}$$

$$F_1 = 2m_0v_0 \cdot \frac{v_0}{2L}$$

$$P_1 = \frac{F_1}{L^2} = \frac{2m_0v_0^2}{2L^3}$$

Force of all molecules

$\frac{1}{3}$ in x direction

$\frac{1}{3}$ in y, $\frac{1}{3}$ in z

in x direction $= \frac{n_0L^3}{3}$

$$\therefore \text{ total pressure} = \frac{2}{3}\,\frac{m_0\overline{v_0^2}}{2}\,n_0 = p$$

compare to $p = n_0k_BT \Rightarrow \boxed{\frac{1}{2}m_0\overline{v_0^2} = \frac{3}{2}k_BT}$

INTERPRETATION: Kelvin Temp $\propto$ $\underbrace{\text{Average Kinetic Energy}}_{\mu_{KE}}$ (L3) 3

also $p = m_0 n_0 \left(\frac{k_B}{m_0}\right) T = \rho R T$

$\rho = m_0 n_0$ $R = \frac{k_B}{m_0} = 287 \, J\,kg^{-1}\,K^{-1}$ air

we can calculate

$$\overline{V_0^2} = \overline{V_x^2} + \overline{V_y^2} + \overline{V_z^2}$$

$$= 3\overline{V_x^2}$$

and $\frac{1}{2} m \overline{V_0^2} = \frac{3}{2} k_B T$

$$\therefore \overline{V_0^2} = 3 \frac{k_B T}{m_0} = 3 R T$$

$$\therefore \overline{V_x^2} = RT = \sigma_{v_x}$$

$$V_{\text{Root mean square}} = \sigma = \sqrt{RT}$$

$$P(v_x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-v_x^2/2\sigma^2}$$

3 D pdf : $\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 e^{-(v_x^2 + v_y^2 + v_z^2)/2\sigma^2}$

$$P(v_x)P(v_y)P(v_z)\,dv_x dv_y dv_z = \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 e^{-v^2/2\sigma^2} \underbrace{dv_x dv_y dv_z}_{\text{vol. element}}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$\hookrightarrow 4\pi v^2 dv$ spherical coord's.

Distribution of molecular speeds v

$$P(v)\,dv = 4\pi \left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^3 v^2 e^{-\frac{v^2}{2\sigma^2}}\,dv$$

MOLES & etc.

(L3) #5

Molecular Wt. : sum of atomic wts

ISOTOPES eg. ^1H , ^2H , ^3H ← tritium

↑ deuterium

$$\underbrace{\qquad\qquad}_{M_H = 1.00794}$$

Species	M
Ar	39.948
N_2	28.0134
O_2	31.9988
H_2O	18.0153

grams versus kg

Most use the gram molecular wt. M_G ← for a particular molecule

Avagadro's Number : # molecules in a gram mole

$$N_A = 6.023 \times 10^{23} \text{ molecules}(\text{mol})^{-1}$$

$$\nu = \#\text{ moles} = \frac{N}{N_A}$$

$$m_0 = \frac{M_G}{N_A} = \text{mass of molecule in grams}$$

$$\left(\frac{M_G}{N_A}\right)\frac{1}{1000} = \text{mass of molecule in kg.}$$

$$p = n_0 k_B T = \underbrace{n_0 m_0}_{\rho} \underbrace{\left(\frac{k_B}{m_0}\right)}_{R_G} T = \rho R_G T$$

$R_G \leftarrow$ for a particular gas G

(use m_0 in kg)

$$p = \rho R_G T = \cancel{\nu N_A} \; n_0 m_0 R_G T \; , \quad \begin{cases} n_0 = \dfrac{\nu N_A}{V} \\[2ex] m_0 = \dfrac{\tilde{M}_G}{N_A} \; , \quad \tilde{M}_G = \dfrac{M_G}{1000} \end{cases}$$

$$p = \left(\frac{\nu N_A}{V}\right) \cdot \left(\frac{\tilde{M}_G}{N_A}\right) R_G T$$

chemistry form

$$p = \frac{\nu}{V} R^* T \; , \quad R^* = \tilde{M} R_G = \text{Universal Gas Const.}$$

$$R^* = 8.31 \; J (mol)^{-1} K^{-1}$$

Also $R^* = k_B N_A$