

Weather Observation and Analysis

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Course Notes

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Chapter 14. VORTICITY

14.1 Curl

Like divergence and gradient, curl involves derivatives of the components of a vector. Like gradient, curl is a vector. The mathematical way of writing the curl of some vector \vec{v} is

$$\nabla \times \vec{v}$$

Even if the vector \vec{v} is entirely horizontal, the curl is a fully three-dimensional vector.

The definition of curl (in three dimensions) is most clearly written in the form of a determinant, as follows:

$$\nabla \times \vec{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

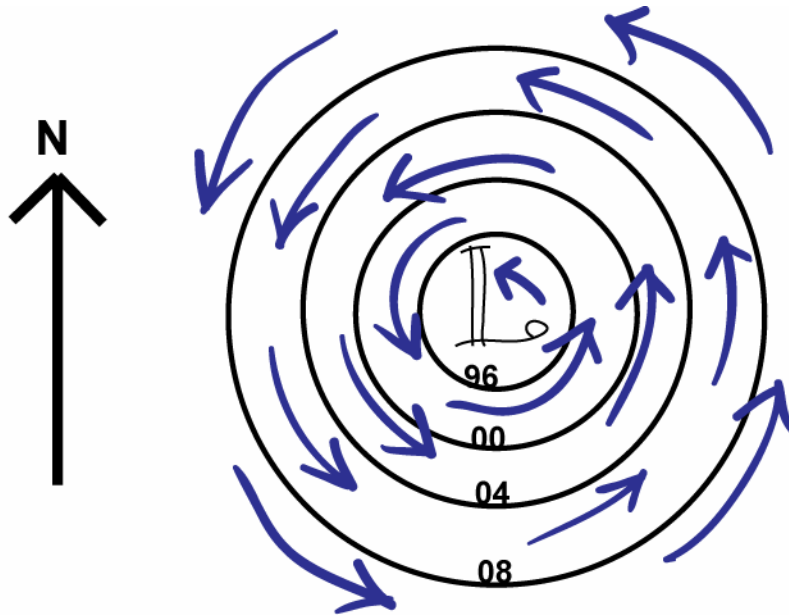
Here we have explicitly assumed that the vector in question is the velocity, so the three velocity components appear on the bottom row.

If you know what a determinant is, great. If you don't know what a determinant is or how to compute in three dimensions, don't worry. You can get by with knowing what the three components of the curl are:

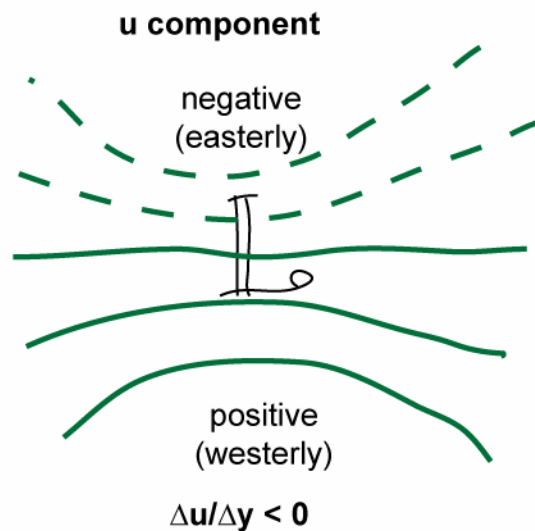
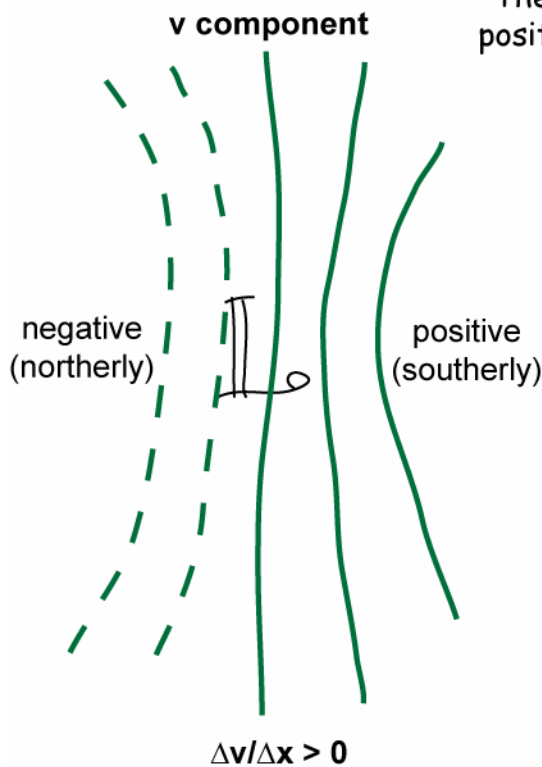
$$\nabla \times \vec{v} = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) \mathbf{i} + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) \mathbf{j} + \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \mathbf{k}$$

Why do they call it the curl? Because it measures the tendency of the vector field (in this case, the velocity) to rotate. Consider, for

example, the vertical component of curl, the last term in the preceding equation. For an entirely two-dimensional world, this is the only component of curl that is non-zero. Suppose now that you are looking down on a low pressure center, with its associated cyclonic (counterclockwise) circulation. If the wind is in geostrophic balance, we know that the divergence is zero, but what about the curl?



The vertical component of vorticity is positive in the case of counterclockwise circulation around a low center.



$$\Delta v / \Delta x - \Delta u / \Delta y > 0$$

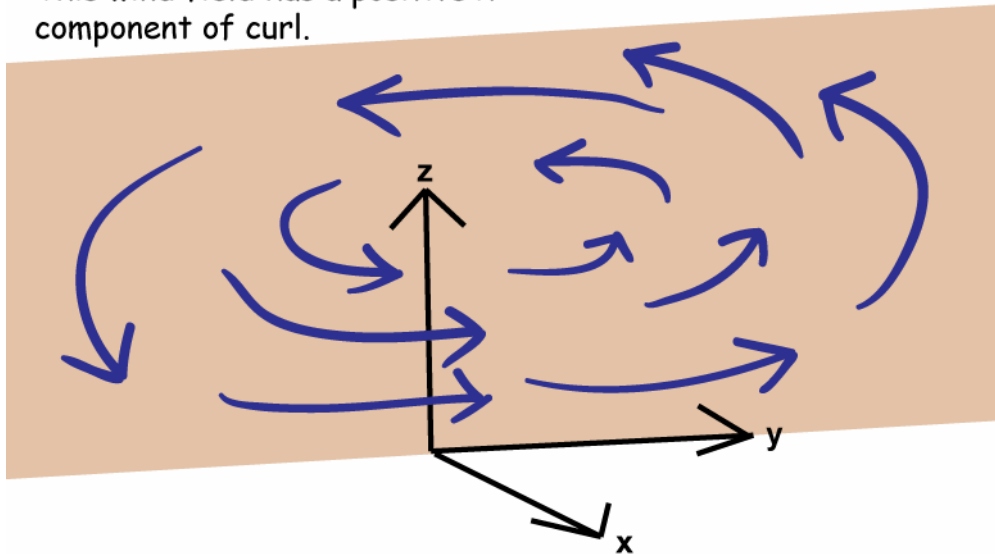
To proceed, let's check out the sign of the vertical component of curl. East of the circulation center, the wind ought to be from south to north (a positive v), while west of the circulation center, the wind ought to be from north to south (a negative v). So v changes in the x direction; specifically, it increases with increasing x . That means that the derivative of v with respect to x is positive.

North of the circulation center, the wind should be blowing from east to west, while south of the circulation center, the wind should be blowing from west to east. So to the south u is positive and to the north u is negative. Thus the derivative of u with respect to y is negative, since u decreases with increasing y .

Taking stock, we have the vertical component of curl equal to a positive term minus a negative term. Since minus a minus is a plus, the vertical component of curl must be positive overall.

You can confirm the same thing for the other two components. Imagine that you're looking at the $x = 0$ plane from a position on the positive x side. Suppose you see counterclockwise rotation of the wind. If you work out the signs of the various derivatives, you'll find that the x component of curl is positive. Repeat the procedure with counterclockwise rotation in the y plane, and you get a positive y component of curl.

This wind field has a positive x component of curl.



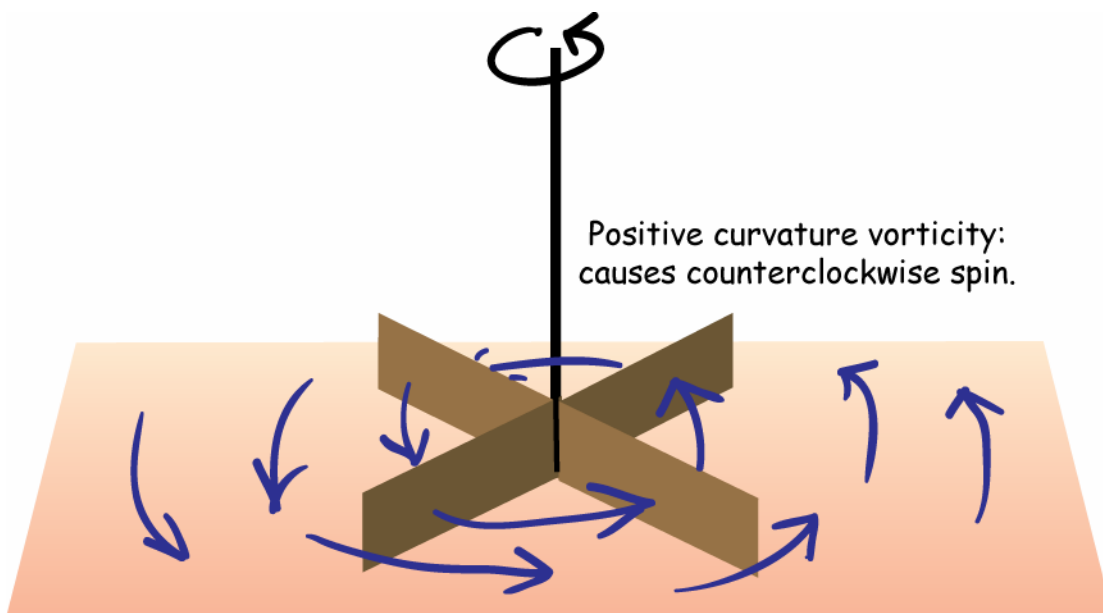
To remember all this, go back to the gig 'em rule. Point the thumb of your right hand in the direction of the component of curl. The vector pattern that produces a curl component in that direction is given by the fingers of your right hand if you are forming the shape of a gig 'em. The vectors are oriented (or the wind is blowing) along your fingers, pointing from the hand along the curve of the fingers to the tips of your fingernails.

14.2 Vorticity

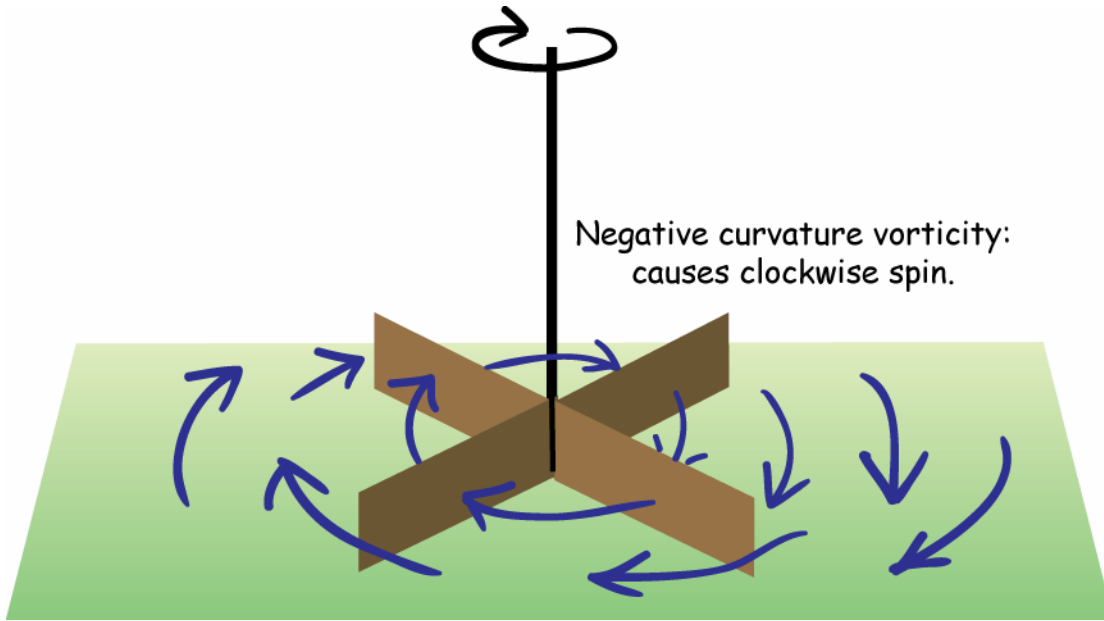
So far we've used the term curl exclusively. In meteorology, or in fluid mechanics in general, the curl of the wind or flow field (that is, the curl of the velocity) gets a special name: the vorticity.

Meteorologists are actually a little bit lax about the term. When they say "vorticity", they are generally referring only to the vertical component of the curl of the wind. When the full three-dimensional curl is meant, meteorologists generally refer to the "vorticity vector". We will follow this convention here. For the rest of this chapter, the vorticity means the vertical component of the curl of the velocity.

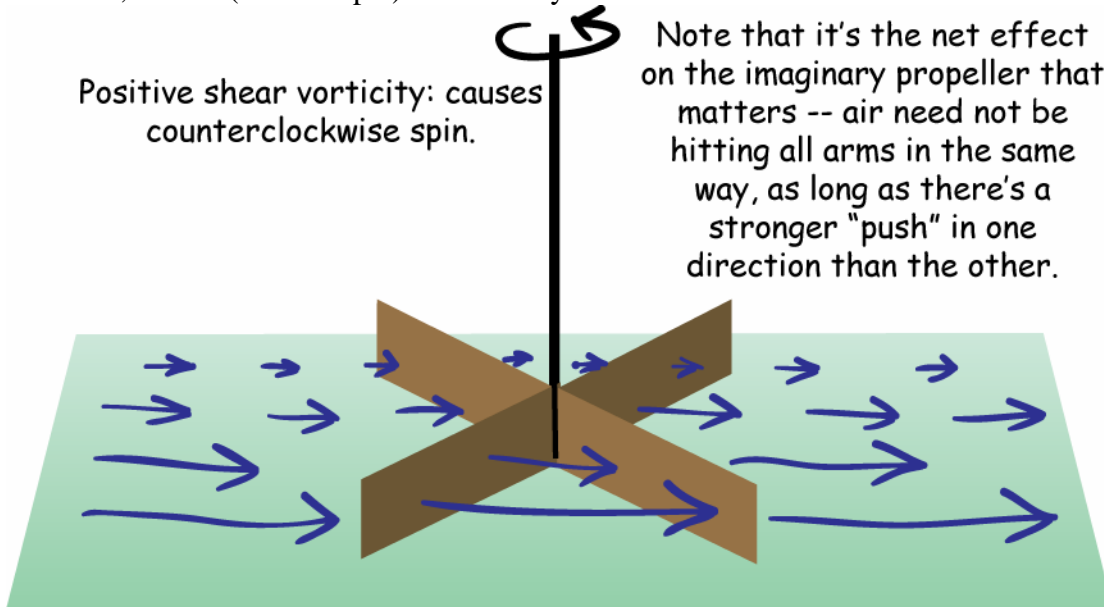
Vorticity has a crucial physical interpretation. The vorticity is a measure of the "spin" of the wind about a vertical axis, with counterclockwise spin being positive. Imagine you stick a four-pronged propeller into the air, with the spin axis of the propeller vertical. We'll take the Cartesian coordinate system as being oriented so that the four prongs are sticking directly north, south, east, and west, respectively. Think about the two prongs sticking east and west. For the wind to try to cause them to spin counterclockwise, you'd want the northward component of wind to be larger on the east prong than on the west prong. In other words, the derivative of v in the x direction would need to be positive, and the larger the derivative, the stronger the turning impulse. As for the two prongs sticking north and south, to get them to turn counterclockwise you'd want the eastward component of wind to be larger on the south prong than the north prong. So the derivative of u in the y direction would need to be negative, and the more negative the better. Thus, the larger the vorticity, the greater the tendency of the wind to spin a propeller.



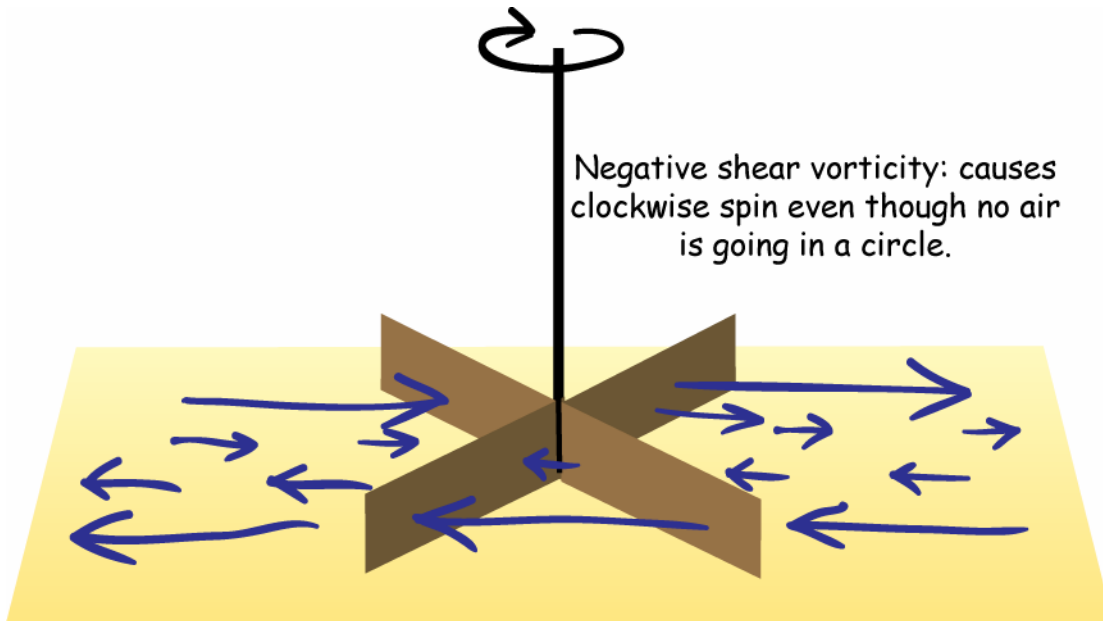
There are lots of different wind patterns that would cause a propeller to spin and that therefore have vorticity. We've considered only one example so far, that of wind rotating in a circle. Vorticity that results from horizontal variations in the direction of the wind is called *curvature vorticity*. For positive curvature vorticity, the wind must be curving counterclockwise, or toward the left facing downwind. Negative curvature vorticity would have the wind curving in the other direction.



The other type of vorticity is called *shear vorticity*. Here, the shear refers to horizontal shear rather than the vertical shear we discussed in the last chapter. Imagine a perfectly straight wind field, and let's say it's blowing from west to east. If the wind speed changes in the north-south direction, so that (for example) the westerly winds to the north are weaker



than the westerly winds to the south, we have horizontal shear. But notice that this situation corresponds to one of the propeller cases, so that the winds on the north and south prongs of the propeller would act to cause the propeller to turn. The north-south wind, being zero everywhere, has no effect, so we have a counterclockwise-rotating propeller and therefore we must have positive vorticity. The basic rule is that if you are facing downwind and the winds are weaker to your left (and stronger to your right), you have positive shear vorticity. If winds are weaker than your right, the shear vorticity is negative.

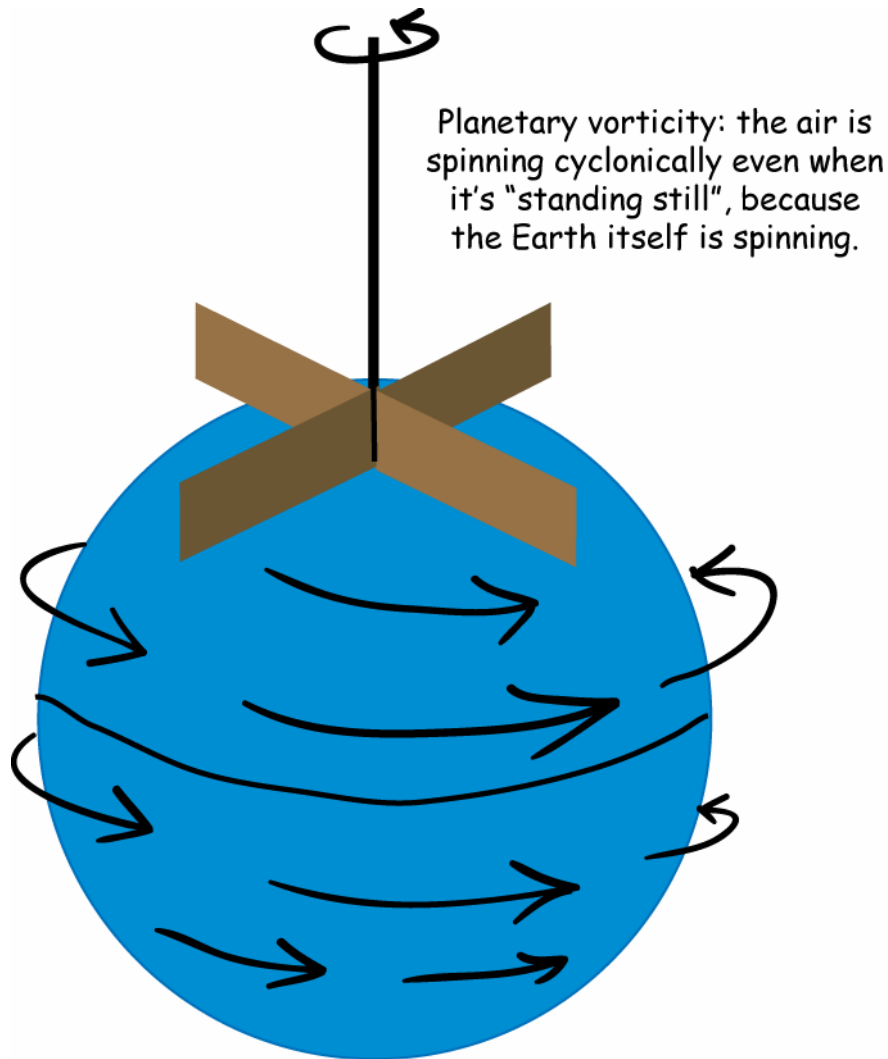


The total (vertical component of) vorticity is the sum of the shear vorticity and the curvature vorticity. Sometimes you have both simultaneously; sometimes they add together and sometimes they cancel.

The atmosphere, even if the wind is not blowing, is rotating counterclockwise (in the Northern Hemisphere). That's because the Earth itself is rotating, and the atmosphere is rotating with it.

Including the effect of the Earth's rotation gives us the *absolute vorticity*. The absolute vorticity has two contributing terms: the vorticity associated with the wind and the vorticity associated with the spin of the Earth. We call these two contributing terms the *relative vorticity* and the *planetary vorticity*.

The planetary vorticity is exactly equal to the Coriolis parameter f . The relative vorticity is usually represented as the Greek letter ζ . So the absolute vorticity is $\zeta+f$.



14.3 Vorticity and Convergence

There are several reasons to care about vorticity. One simple reason is that high vorticity implies strong cyclonic circulation centers or strong troughs. If cyclonic vorticity is concentrated along a line, it marks the windshift associated with a front. (Fronts always have cyclonic vorticity.) But perhaps the most important reason is related to the relationship between vorticity, convergence, and vertical motion, which lets us diagnose the intensification of cyclonic circulation centers or troughs, and their converse, anticyclonic circulation centers or ridges.

It would be great if we could diagnose changes in pressure directly. Since pressure is proportional to the weight of air in the overlying air column, one might think there's some hope of diagnosing temperatures and thereby diagnosing pressures. But that's not very workable, because horizontal temperature advection and vertical motion tend to act to oppose each other, and the net temperature change is difficult to determine.

We can also diagnose the forces acting on the wind, and thereby diagnose changes in the wind. But to do that, we would need to be able to predict changes in the pressure gradient force, so we have the same problem as if we're trying to diagnose pressure.

Vorticity, however, is fairly easy to diagnose. There are two basic techniques, the first involving convergence and vertical motion, the second involving something called potential vorticity. First, we'll talk about the connection between vorticity, convergence, and vertical motion.

In Chapter 9, the connection between convergence, divergence, and vertical motion was first discussed, and they go hand in hand. A brief review: if there's large-scale upward motion, there must be convergence in the lower troposphere and divergence in the upper troposphere. Conversely, if there's large-scale downward motion, there must be divergence in the lower troposphere and convergence in the upper troposphere.

Now for the connection between all this and vorticity:

$$\frac{D\zeta}{Dt} = -(f + \zeta)(\nabla_h \cdot \vec{v}) + \text{other stuff}$$

This equation states that the rate of change of vorticity following an air parcel is proportional to the absolute vorticity times the negative of the divergence (i.e., the convergence), plus some other stuff. The "other stuff" would be rather ugly if I wrote it all out, but fortunately, the larger the horizontal scale, the smaller the other stuff. Once we get to weather systems larger than a few hundred kilometers or so, the other stuff is completely negligible and the divergence and convergence controls the vorticity.

Consider changes to vorticity in the lower troposphere. If there's a low pressure center, it will necessarily have cyclonic relative vorticity, which is positive. If the low intensifies, the vorticity must increase. The above equation states that, for the vorticity to increase, there must be convergence and therefore aloft there must be upward motion. If, instead, the low weakens, there must be divergence present and downward motion aloft.

High pressure centers have anticyclonic relative vorticity. This is negative in the Northern Hemisphere, but the absolute vorticity (the sum of the relative vorticity ζ and the planetary vorticity f) will almost always be positive in the Northern Hemisphere. So, for the high pressure center to intensify, the vorticity must decrease. Hence there must be divergence present, and therefore downward motion present aloft.

Notice that if the issue is the intensification or weakening of a cyclonic disturbance (a trough) in the upper troposphere, you still need convergence, but since that convergence would be taking place in the

upper troposphere it would mean downward motion in the middle troposphere.

The basic rule is as follows: mid-tropospheric upward motion implies that the relative vorticity will become more positive (or less negative) at low levels and less positive (or more negative) aloft, and mid-tropospheric downward motion implies that the relative vorticity will become more negative (or less positive) at low levels and less negative (or more positive) aloft.

14.4 Predicting Vertical Motion

Using the principles described in the previous paragraph, we can use vertical motion to diagnose intensification and weakening of weather systems. All that's left is to predict vertical motion!

In past chapters, we've encountered several indicators of vertical motion. One is friction: friction produces cross-isobar flow and convergence within lows, while the cross-isobar flow is outward and divergent around highs. However, frictionally-induced vertical motion is the one kind of vertical motion that doesn't lead directly to intensification of large-scale weather systems, because the friction that causes the vertical motion also slows down the wind. (That's one of the things hidden in the "other stuff".)

Another is the effect of curvature and ageostrophic winds around upper-level troughs and ridges. Upward motion is expected downstream of troughs and upstream of ridges, while downward motion is expected downstream of ridges and upstream of troughs. So, for example, if a low-level cyclone is located downstream of a trough, one would expect to find upward motion and thus low-level convergence and intensification of the cyclone.

Another is the effect of accelerations within a jet streak. Upward motion is expected beneath the right-hand side of the entrance region and beneath the left-hand side of the exit region, while downward motion is expected beneath the left-hand side of the entrance region and beneath the right-hand side of the exit region.

Another is the effect of intensification or weakening of fronts. If a front is becoming stronger, there should be upward motion on its warm side, while if a front is weakening, there should be upward motion on its cold side.

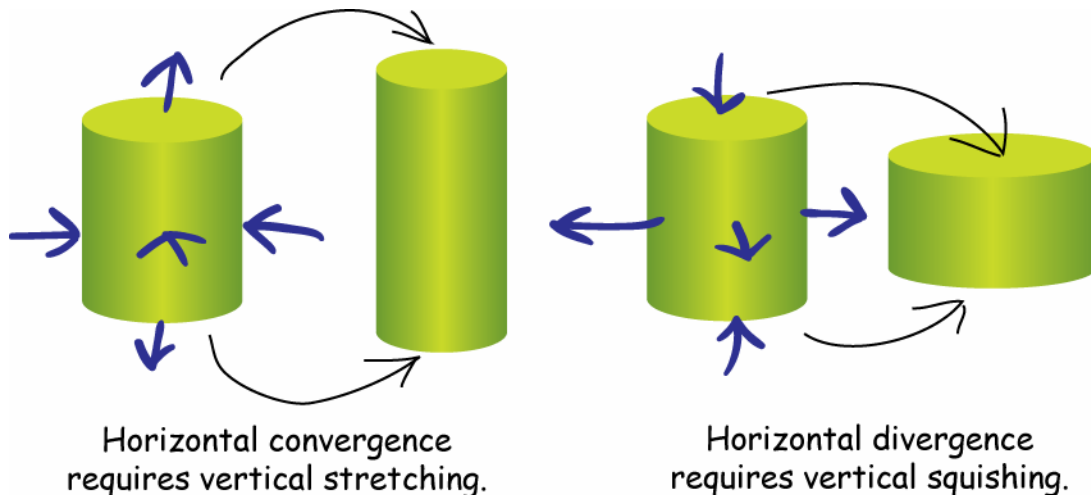
Another is the effect of topography, with upslope and downslope flow. Except here is where the simple rule about low-level convergence being associated with upward motion doesn't apply. The effect of topographic upward and downward motion will be much easier to explain in the context of potential vorticity later in this chapter.

Rather than going through a laundry list of possible mechanisms, it would be nice to have a single equation that relates vertical motion to other straightforward, easily observable aspects of the atmospheric state. Such an equation does exist. It's called the omega equation, after the symbol for vertical motion in pressure coordinates. There are various forms of the equation, involving different ways of arranging the terms, different simplifications and cancellations. We won't go into the details here, but the physics behind it is a three-dimensional analog to the vertical motion associated with frontogenesis. Basically, anything that changes the strength or orientation of the vertical wind shear or horizontal temperature gradient experienced by an air parcel requires vertical motion to keep everything in thermal wind balance.

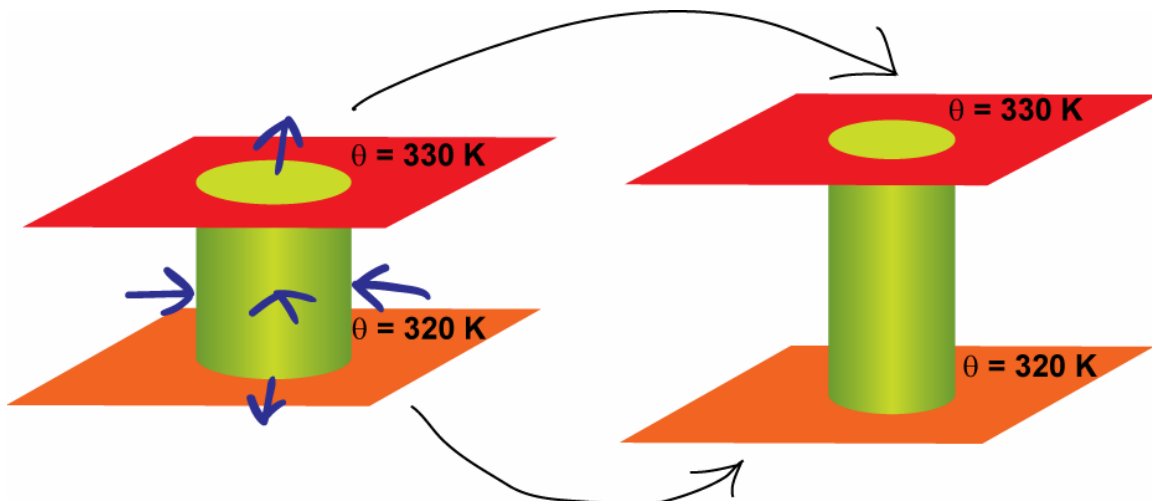
Until it's time to study the omega equation, it will be sufficient to utilize the list of patterns and processes associated with vertical motion.

14.5 Potential Vorticity

The close relationship between vorticity changes and divergence/convergence suggests that the vorticity of a particular air parcel depends entirely on its history of divergence and convergence. If we now imagine that air parcel as being three-dimensional, horizontal convergence would squish it horizontally and stretch it vertically. Similarly, horizontal divergence would stretch it out horizontally and squish it vertically. In principle, if you had a way of determining how tall an air parcel is, you could tell how much vorticity it had.



There is such a way. Remember that potential temperature is conserved by air parcels, and increases upward. Thus, the potential temperature at the top and bottom of an air parcel stays the same, even as the air parcel stretches and squishes. Changes in the vertical spacing of isentropic surfaces (surfaces of constant potential temperature) corresponds to changes in the vertical extent of air parcels.



Since air parcels conserve potential temperature, vertical stretching (or squishing) of air parcels corresponds to increasing (or decreasing) the vertical separation between isentropic surfaces.

The vertical spacing of isentropic surfaces has a name: the stratification. If the isentropic surfaces are close together, the vertical derivative of potential temperature with respect to height or pressure is large and the stratification is high. If the isentropic surfaces are far apart, the vertical derivative of potential temperature is small and the stratification is weak.

So if an air parcel undergoes horizontal convergence, its absolute vorticity will increase and its stratification will decrease. Conversely, if an air parcel undergoes horizontal divergence, its absolute vorticity will decrease and its stratification will increase.

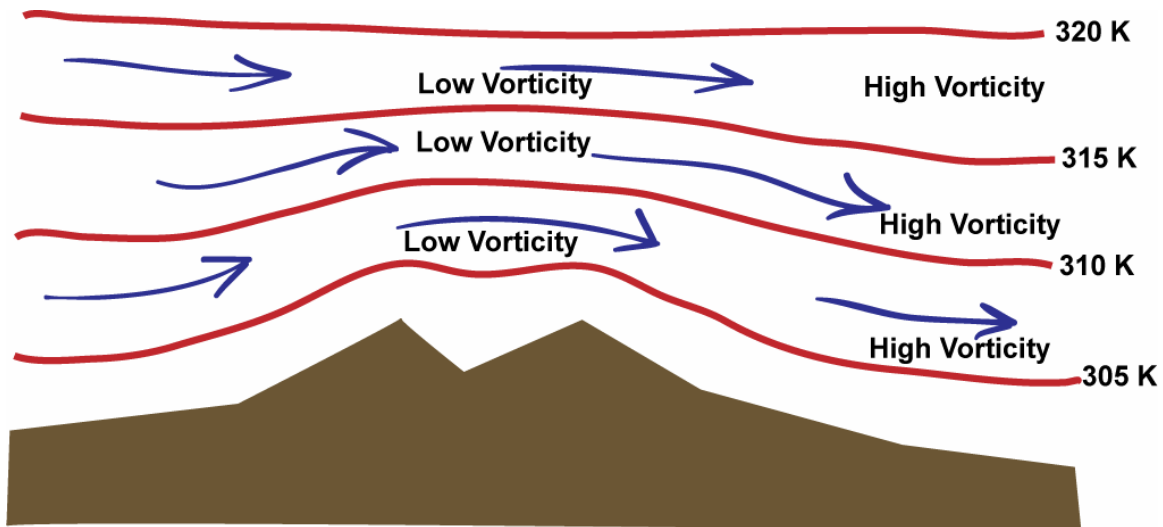
It takes a fairly simple bit of mathematics to show that the product of vorticity and stratification stays the same: they go up and down proportionally so that their product is constant. This product is known as the potential vorticity, which we will write as P :

$$\frac{DP}{Dt} = \frac{D}{Dt} \left(-g\zeta \frac{\partial \theta}{\partial p} \right) = 0$$

The potential vorticity is conserved under the following conditions: (1) no friction; (2) no diabatic heating, such as latent heating. Above the ground, there's no friction, and if there's no cloud either, there's no significant diabatic heating. So P is conserved most of the time. Also, strictly speaking, the vertical vorticity in P should be computed from the derivatives of wind on the isentropic surfaces, rather than on constant pressure or height surfaces.

One simple application of potential vorticity is in flow over topography. Picture air flowing over a mountain range. As air goes up

and over, so do the isentropic surfaces. Since potential vorticity is constant, the horizontal variations of vertical spacing of isentropic surfaces can be used to infer the places where the absolute vorticity is large or small. Over the mountains, the isentropic surfaces are necessarily closer together and the absolute vorticity is smaller. On the lee (downwind) side, the air parcels stretch and the absolute vorticity increases. Most of the time, there's a cyclone in the lee of large mountain ranges for this reason.



As air flows over mountains, air parcels get squished and their vorticity decreases. As it descends the lee side of the mountains, the air parcels are stretched vertically and their vorticity increases. Thus, cyclones weaken as they pass over mountains and intensify again on the lee slopes.

One thing to be careful of: diagnosing vorticity directly this way only works within a single isentropic layer. For instance, in the example shown above, if the 315-320 K isentropic layer was shallower, the vorticity in that layer wouldn't necessarily be lower. All we can say is that, within a given isentropic layer, the vorticity decreases as air goes up the mountain range and increases as the air comes back down.

14.6 Potential Vorticity and Vertical Motion

Potential vorticity isn't just useful for understanding lee cyclogenesis. More generally, it's the most straightforward way of understanding the dynamics of the atmosphere. The principles of potential vorticity dynamics are listed here:

(1) Potential vorticity is mostly conserved. (You knew that one already.)

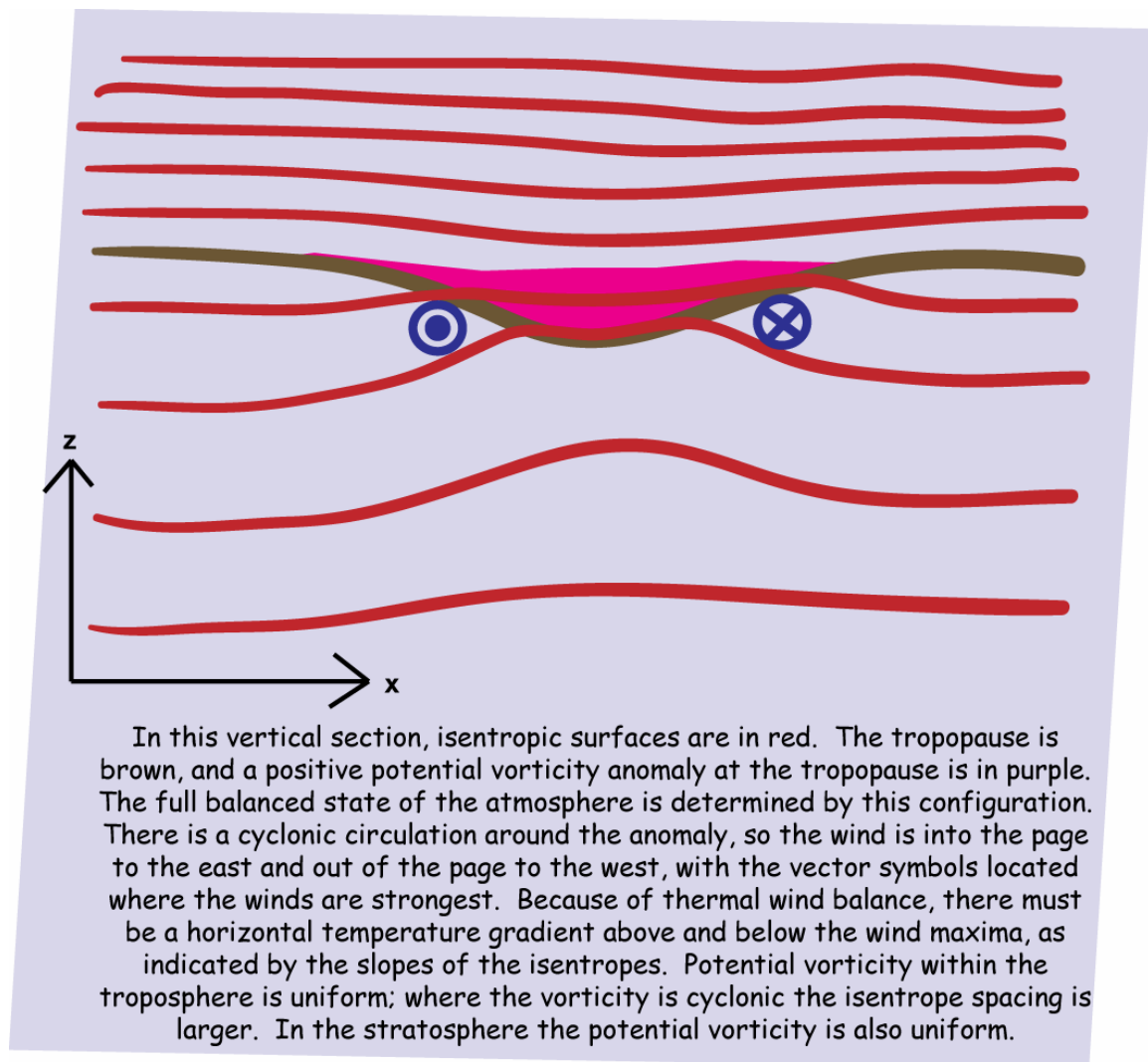
(2) Anywhere the potential vorticity is larger than most of what surrounds it at the same isentropic level is a *positive potential vorticity anomaly*. If the potential vorticity is smaller than what surrounds it at the same isentropic level, it's a *negative potential vorticity anomaly*.

(3) If the winds are balanced, a single positive potential vorticity anomaly is at the center of a cyclonic circulation; a negative anomaly is at the center of an anticyclonic circulation. The winds weaken away from the anomaly in all three dimensions.

(4) If there are multiple anomalies, the effects of all the anomalies can be added together to get the overall wind pattern.

(5) The stratosphere, being very stably stratified, has much higher potential vorticity than the troposphere.

(6) Anomalies of potential temperature near the ground have the same effect as potential vorticity anomalies in the interior of the troposphere.



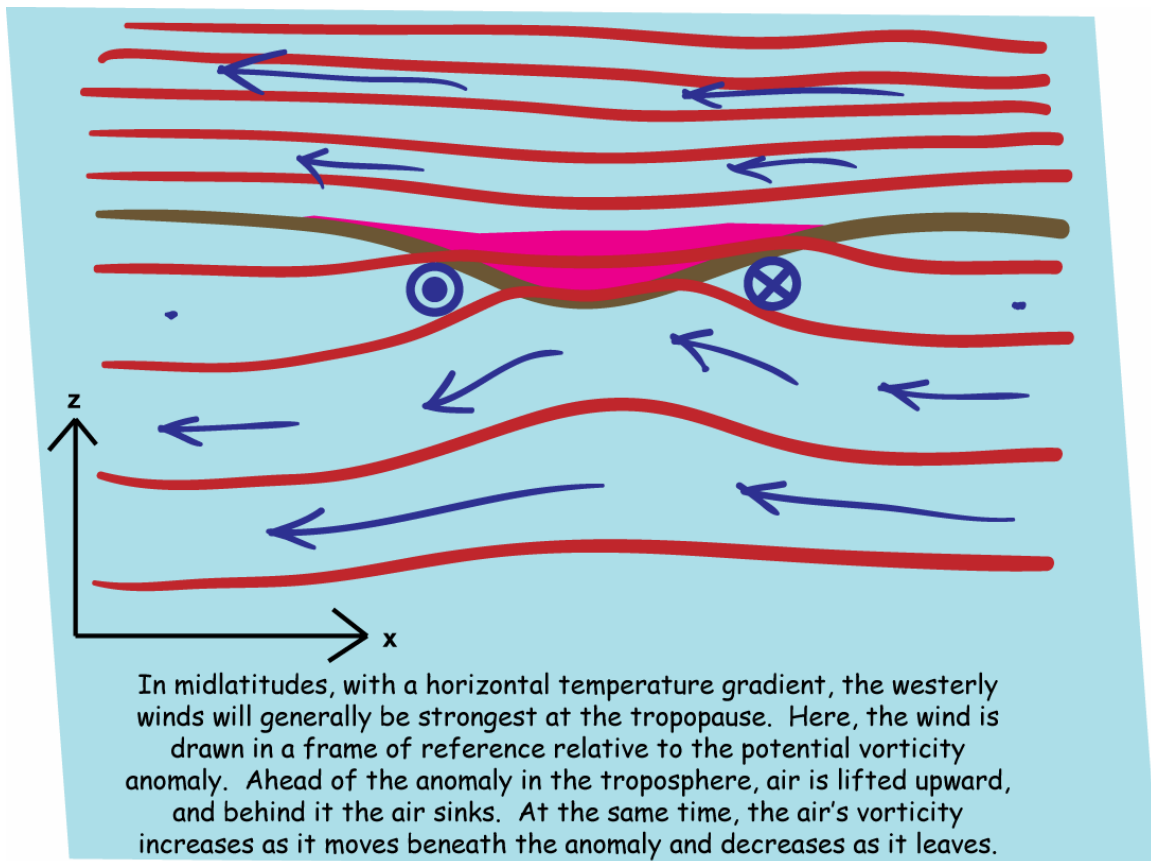
In item (3), “balanced” means geostrophic and hydrostatic balance, and therefore thermal wind balance as well. Winds, pressures, and temperatures are all consistent with each other. A given potential vorticity distribution constrains the winds, pressures, and temperatures to have a

particular pattern as well. Even the isentropic surfaces are constrained to have a particular pattern by the particular arrangement of potential vorticity. And you can't get a cyclone or anticyclone, a trough or a ridge, without a potential vorticity (or near-ground potential temperature anomaly) at its core.

We'll now apply these principles to diagnosing vertical motion. The key is the potential temperature surfaces. Since potential temperature is conserved, air that starts out on a particular isentropic surface can never leave it. Since the isentropic surface positions are determined by the potential vorticity distribution, air must move up or down along isentropic surfaces if they are sloped.

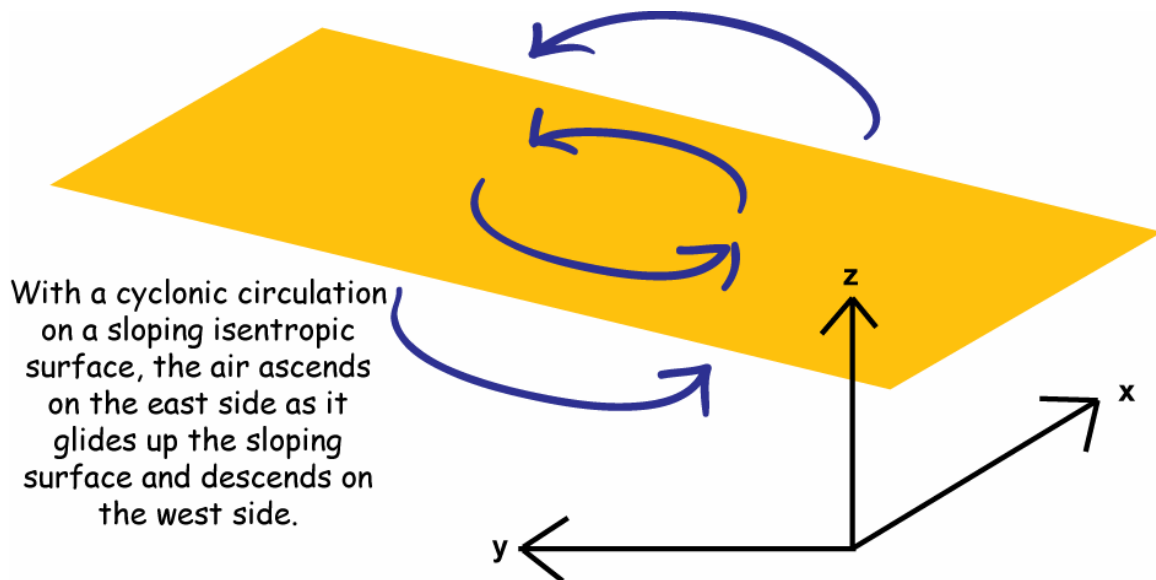
Suppose there is an upper-level trough. Since a trough is cyclonic, there must be a positive potential vorticity anomaly associated with it. That anomaly is generally located at the tropopause, as shown in the previous figure. With just the one potential vorticity anomaly, in an otherwise undisturbed environment, there would be no vertical motion. Even though many of the isentropic surfaces have slopes, the air is just circulating around the potential vorticity anomaly without going up or down.

The situation changes if the potential vorticity anomaly is embedded in a sheared environment. Usually, the temperature decreases



toward the pole in midlatitudes, so this is the ordinary situation. Because there's a horizontal temperature gradient, there's also vertical wind shear as shown in the figure on the previous page. Essentially, the tropopause potential vorticity anomaly is zooming eastward past most of the rest of the atmosphere. As the anomaly approaches, air ahead of it in the troposphere must be lifted upward because the isentropic surfaces slope upward there. As the anomaly goes by, the air descends again. This process has been called the "vacuum cleaner effect" because it's as though the potential vorticity anomaly is sucking the isentropic surfaces and the air toward it.

So far we've described the vertical motion caused by the interaction between the large-scale shear and the bumps in the isentropic surfaces due to the anomaly. There's also vertical motion caused by the interaction between the large-scale sloping isentropic surfaces and the winds due to the anomaly, as follows: since the isentropic surfaces in the troposphere must slope upward toward the north (because of the horizontal temperature gradient), northerly winds are associated with downward motion and southerly winds are associated with upward motion. With a cyclonic potential vorticity anomaly, the air ahead (to the east) of the anomaly is moving northward and ascending, and the air behind the anomaly is moving southward and descending.



To summarize, once the relative motion of the air and the isentropic surfaces are known, it is easy to determine where the air is gliding up and where it is gliding down.

Notice that both the convergence/divergence technique and the potential vorticity technique predict that there should be upward motion ahead of an upper-level trough and downward motion behind an upper-level trough. This is to be expected: if the two techniques gave

contradictory answers, one of them would be wrong. In practice, meteorologists use whatever approach is easiest and least ambiguous.

Questions

1. Suppose u and w are both zero in a particular area, but v increases toward the south at the rate of 1 m/s per 10 km and increases with height at the rate of 10 m/s per km. What is the curl of the velocity? Express the answer as a vector.

2. Sketch a vertical section for which the y component of the curl of the velocity field is negative.

3. Sketch wind fields with the following characteristics: a) Anticyclonic shear vorticity with the u component of horizontal wind zero everywhere. b) Cyclonic curvature vorticity and anticyclonic shear vorticity.

4. At 50 N, the convergence is $1.5 \times 10^{-1} \text{ s}^{-1}$ and the relative vorticity is $3.2 \times 10^{-5} \text{ s}^{-1}$. Is the relative vorticity increasing or decreasing? How rapidly?

5. One PVU (potential vorticity unit) equals $1 \times 10^{-6} \text{ m}^2 \text{ K s}^{-1} \text{ kg}^{-1}$. Show that the potential vorticity in the troposphere should be somewhere near 1 PVU.

6. Sketch a vertical section with isentropes and winds consistent with a low-level positive potential temperature anomaly.

7. What part of the atmosphere in the vicinity of a negative potential vorticity anomaly is a warm core anticyclone?