Weather Observation and Analysis

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Course Notes

Chapter 9. VERTICAL MOTION

9.1 Divergence in Two and Three Dimensions

The gradient symbol, encountered earlier in these notes, is really a vector operator. “Gradient” itself does not have a magnitude and direction, but the gradient *of something* does. A vector operator such as the gradient operator (or “del”, for short) is something that represents a mathematical operation but that can be manipulated like a vector. These manipulations always are performed using the vector components as manipulation tools.

The components of the del operator in three dimensions are

$$\nabla = \mathbf{i}\left(\frac{\partial}{\partial x}\right) + \mathbf{j}\left(\frac{\partial}{\partial y}\right) + \mathbf{k}\left(\frac{\partial}{\partial z}\right)$$

So

$$\nabla T = \mathbf{i}\left(\frac{\partial T}{\partial x}\right) + \mathbf{j}\left(\frac{\partial T}{\partial y}\right) + \mathbf{k}\left(\frac{\partial T}{\partial z}\right)$$

is just the vector formed when each component of del operates on temperature.

Another quantity that can be created through mathematical manipulation of the gradient vector is divergence. The divergence of some vector field, or div for short, is the dot product of del with that vector field. The word divergence itself, when not followed by the clarifying statement of the vector field to which it applies, is assumed to mean the divergence of the vector velocity.

Remember from Chapter 7 that the dot product of two vectors can be expressed as the sum of the products of each of the components. When dotting something with the del operator, each component of del operates on the corresponding components of the other vector. Thus the divergence of a vector is the derivative with respect to *x* of the *x* component, plus the derivative with respect to *y* of the *y* component, plus (if the vector is three-dimensional) the derivative with respect to *z* of the *z* component.

In three dimensions, using component notation, the divergence of the velocity vector is:

$$\nabla \bullet \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

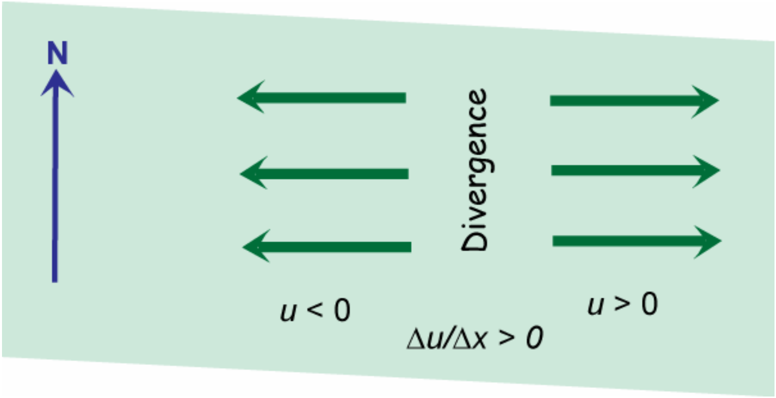
In two dimensions it would be:

$$\nabla_h \bullet \vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

Perhaps you can now see how del works in vector-form equations. For the dot product, each component of del is combined with the corresponding components of the other vector.

Now, how does this mathematically-defined quantity compare to what you would call divergence in normal conversation? That is, if you talk about divergence in normal conversation? It’s the same thing, actually. Imagine a longitude line where, along that line, the wind is calm. Imagine everywhere to the east of the line the wind is blowing from west to east (meaning the *x* component of the wind, *u*, is positive), and everywhere to the west of the line the wind is blowing from east to west (meaning *u* is negative). If you sketch those vectors on a map, it will certainly *look* like divergence, as shown on the next page.

Can you tell from that description whether $\partial u/\partial x$ is positive or negative? It must be positive, according to any of the following three arguments: (1) if you graph *u* as a function of *x*, the graph slopes upward toward the right; (2) if you try to compute the finite difference using values of *u* on either side of the longitude line, you’d be subtracting a negative value from a positive value, and the answer is always positive; and (3) if you analyzed *u* by drawing lines of constant *u* on the map, the lines would run north-south and the gradient vector would point west-to-east toward the higher values.



So $\partial u/\partial x$ is positive, and since the wind is only east-west throughout this level in this imaginary world, $\partial v/\partial y$ is identically zero. Thus, according to the above definition, the two-dimensional divergence is positive. So, yes, we have divergence in both a visual sense and a mathematical one.

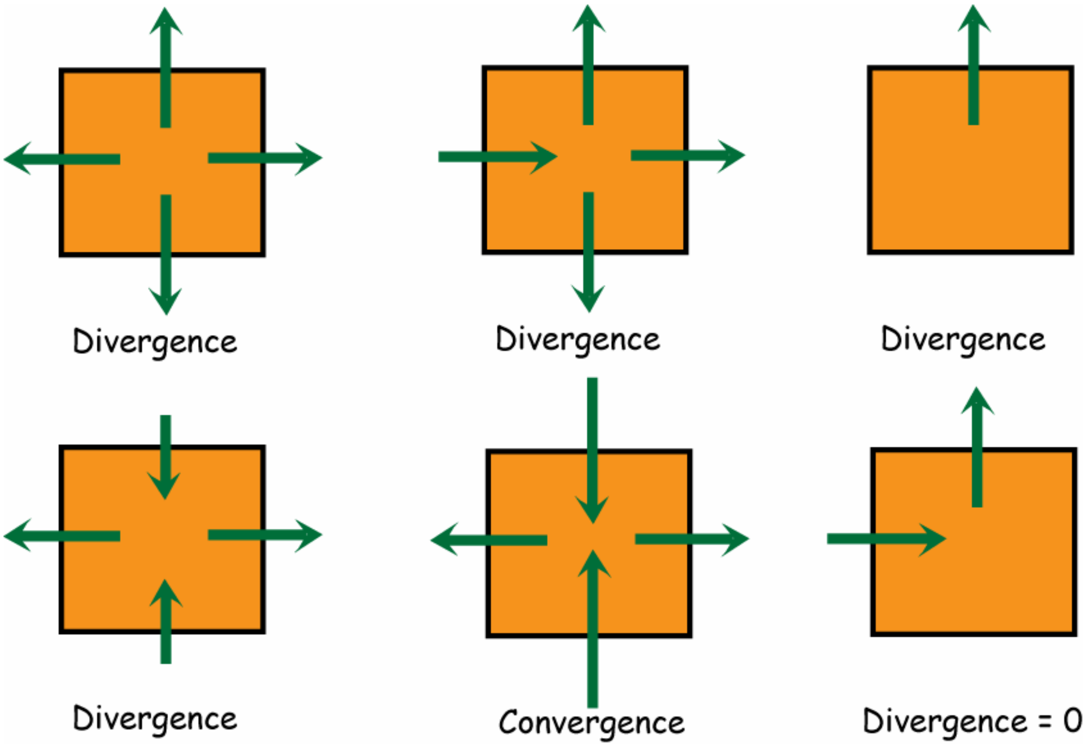
The opposite of divergence in normal conversation is convergence. Mathematically, we don't need a separate definition; we call it "convergence" if the value of the divergence is negative.

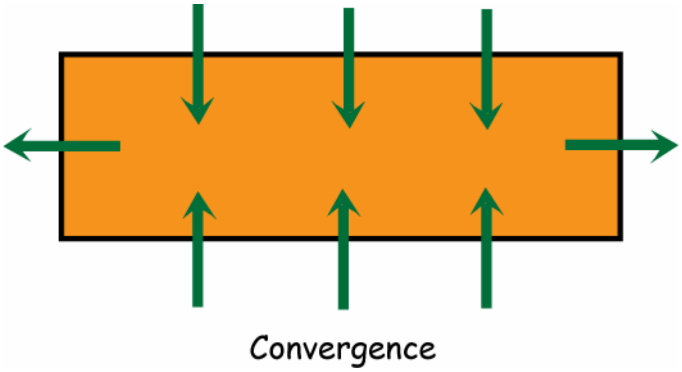
9.2 The Divergence Theorem

There's another handy visual technique for estimating the divergence of the horizontal wind. On a map with winds, draw a box. Now look at the wind going into or out of the box along all four of its sides. If there's more air going out of the box than in, there's divergence. If there's more air going into the box than out, there's convergence.

This technique arises directly out of the mathematical definition of divergence. In comparing the winds going into or out of the box on the left and right sides of the box, you're looking at the *u* component of the wind. Finding whether there's net outflow or inflow means subtracting the *u* component across the left side of the box from the *u* component across the right side of the box: a positive number means net outflow and a negative number means net inflow. You're computing the numerator of $\partial u/\partial x$; to estimate the derivative, you need only divide by the distance between the two sides. Figuring out whether there's net outflow or inflow along the top and bottom of the box involves computing $\partial v/\partial y$.

Suppose you were being perverse and instead of drawing a box, you drew a long rectangle. To compare the amount of air going out the long sides from the amount going out the short sides, you would have to consider the fact that a given outward velocity along a long side means a lot of air leaving the box, while a given outward velocity along the short side doesn't mean much air at all. To do it properly, you might even have to break out the calculator and multiply the outward velocity along the short sides by the length of a short side and the outward velocity along the long sides by the length of a long side, in order to give proper weight to the velocities along the respective sides.





Alternatively, you could compute the two-dimensional divergence directly using finite differences: the difference in the *u* component on either side, divided by the distance between the sides; plus the difference in the *v* component on the top and bottom, divided by the distance between the top and bottom.

The fact that these two techniques are both useful for the same purpose is not an accident. In a sense, they are equivalent, according to the *divergence theorem*. In two dimensions,

$$\iint \left(\nabla_h \bullet \vec{v}\right) dx dt = \oint \left(\vec{v} \bullet \mathbf{n}\right) d\mathbf{l}$$

where **n** is a unit vector pointing outward from the area and *d***l** represents a one-dimensional integral along the edge of the box.

If you haven't dealt with integration yet, particularly multi-dimensional integration, do not panic. Consider the left-hand side first. The integral of something over an area or volume is equal to the size of the area or volume times the average value of that something. Remember, we used a similar approach (in one dimension) in Chapter 8, relating thickness to the average temperature within the layer. So no need to worry about a two-dimensional integration; we can just replace the left-hand side with

$$\iint \left(\nabla_h \bullet \vec{v}\right) dx dt = A\overline{\left(\nabla \bullet \vec{v}\right)}$$

where A is the area of the box.

On the right-hand side of the two-dimensional divergence theorem, as written above, is something called a *line integral*. Again, not to worry. Here's one way to think about a line integral. The value of a meteorological variable is different depending on where on (or above) the Earth that variable is measured. Similarly, the spatial derivative and gradient of that variable changes from location to location. It doesn't make sense to talk about the value of the gradient of a variable unless you say where that value applies.

The same thing applies to integrals. If you're integrating something with respect to x, you have to specify not just the x interval but also the value of y (and z) during the integration. In three-dimensional space, the integral is being carried out along a line parallel to the x axis.

Just like there's no constraint that says that the x axis has to point in a particular direction, there's no constraint that says that integrals have to take place along lines parallel to a particular coordinate axis. You can compute an integral along any arbitrary line in space. In fact, you can integrate around a circle, a box, or any other closed loop. Integrals of this sort are represented by the little circle symbol on the integral symbol. The *d***l** represents the fact that the line along which integration takes place doesn't necessarily correspond to a particular coordinate direction.

In the situation here, the integral is computed along the entire edge of the box, but along each side, it really is just an ordinary single integral. More generally, along each side or line segment, the integral is equal to the average value of the integrand along that segment times the length of that segment. To get the complete line integral, just add up the integrals of the individual segments.

Since **n** is a unit vector locally pointing out of the box along each side, the dot product of that vector with the wind is simply the component of wind blowing out of the box.

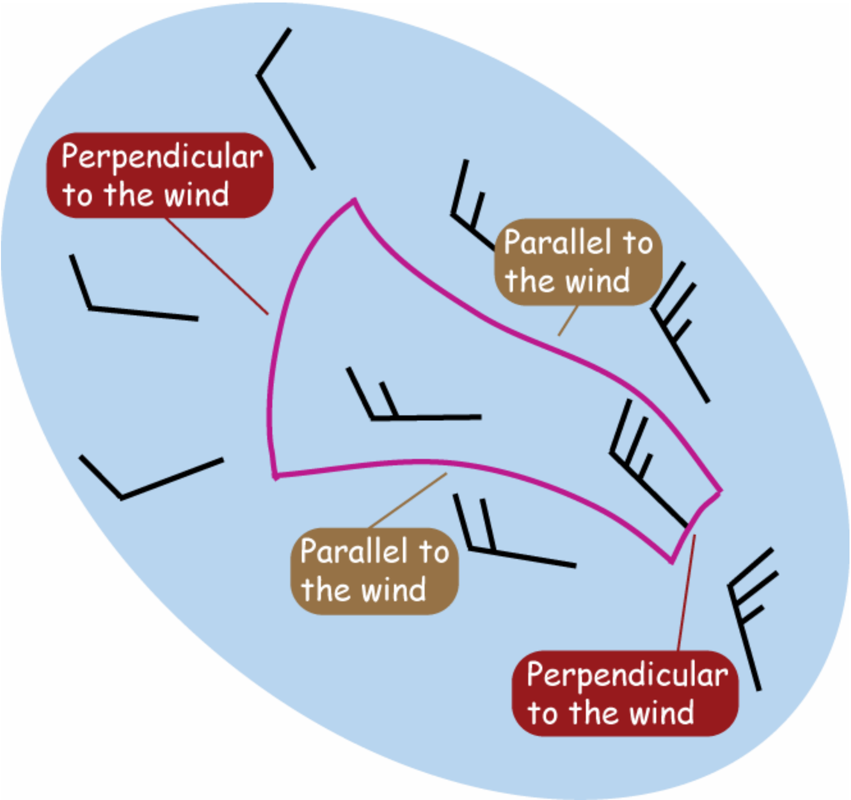
Now we are ready to put the divergence theorem into words. The divergence theorem is saying that the average value of divergence within a given box times the area of the box is equal to the average flow out of the box along each of the sides, times the length of the sides. Or equivalently, the average divergence (times the area) is equal to the average flow out of the box along the entire perimeter, times the length of that perimeter.

9.3 Estimating Divergence on a Weather Map

That's all fine, but it is still rather complicated. To even tell whether you have divergence or convergence, you would have to compute four average outward wind components and (if it's not a perfect square) multiply each by the length of the corresponding sides. Fortunately, there's a shortcut.

This shortcut makes use of the fact that *the area doesn't have to be a four-sided square or rectangle*. Indeed, any closed shape will do. It could be circle, or a parallelogram, or a pentagon, or any arbitrary shape whatsoever. How does this help? Well, suppose you draw a four-sided area with the following properties: two of the sides are exactly parallel to the wind and the other two are exactly perpendicular to the wind. If the wind direction changes, these sides can even be curved, and the certainly don't need to be parallel to each other, as long as they're locally parallel to (or perpendicular to) the wind.

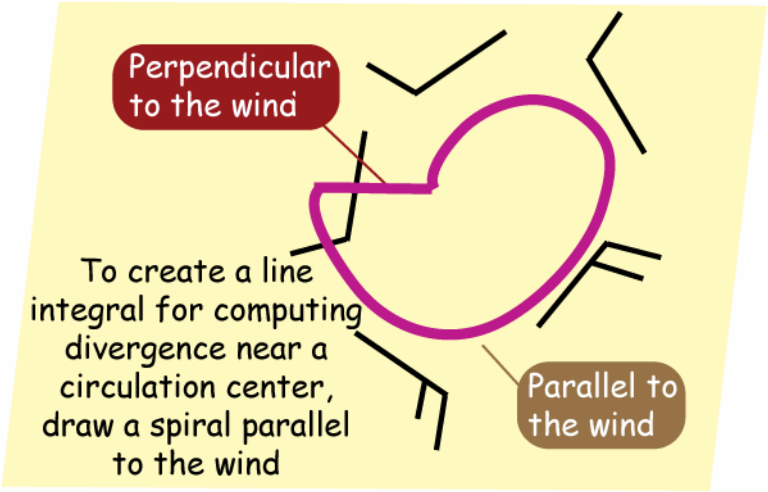
Now with our special four-sided area, two of the sides are parallel to the wind, so along those two sides, the component of wind into or out of the area is exactly and identically zero. So half the computation is done already. Along the other two sides, the wind is directly inward or outward, so the magnitude of the outward component is given by the total wind speed itself, positive for outward and negative for inward. So no need at all to compute components. To compute the full line integral, all that's necessary is to take the average wind speed out of the area times the length of that side, and add it to the average wind speed out of the area along the opposite side, times the length of that side. We've gone from a complicated multidimensional integration to something that can almost be done in one's head!



This approach works well when you have winds all blowing from roughly the same direction. What if you are near the center of a high or low pressure center? In that case, you're better off drawing your area as a spiral that encloses the center. Draw the spiral so that the line is parallel to the wind everywhere, except for one line segment connecting the two ends of the spiral.

In general, at the ground, every circulation center should have net flow inward or outward, because of the effect of surface friction. If you've drawn a circle parallel to the wind everywhere, and it ends at the same place it starts, you may have tried to hard to draw a complete circle. Conversely, if you're analyzing something like surface pressure that must form a closed loop around a circulation center, you should find that the wind tends to blow slightly inward or outward everywhere.

In three dimensions, the divergence theorem works essentially the same way. Now, instead of the margins of an area, you would work with the surfaces of a volume. If there is net flow into a given volume, there is convergence; if there is net flow out of a given volume, there is divergence.



9.4 Three-Dimensional Divergence and the Atmosphere

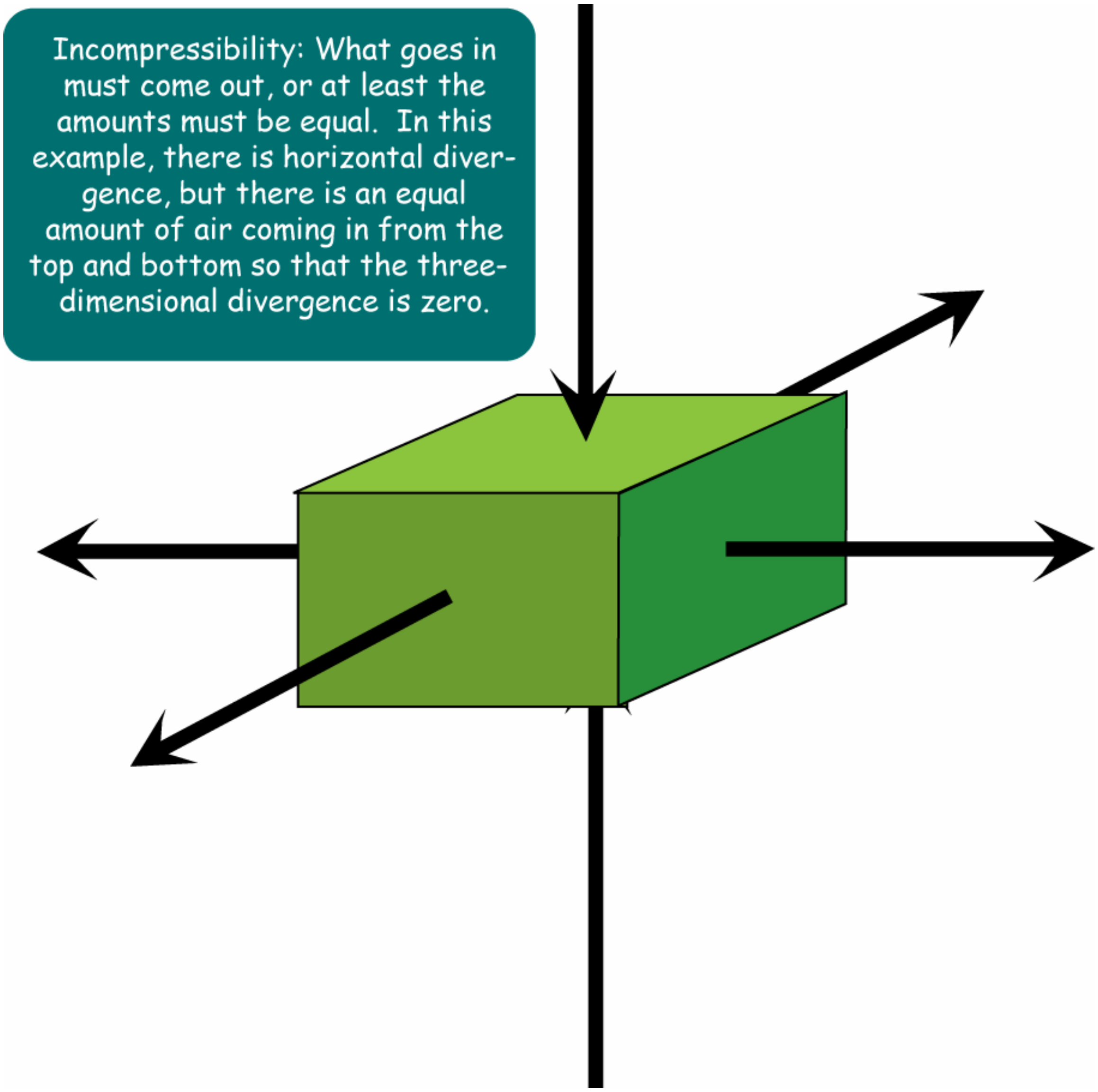
Any arbitrary vector field might have divergence in some places and convergence in others. Wind vectors are special, though, because they actually represent mass traveling into or out of an area (in two dimensions) or volume (in three dimensions).

The laws of physics place a strong constraint on divergence and convergence in the atmosphere. In particular, the law of mass conservation states that if you have net divergence of mass from a volume, the density within the volume must decrease. Written mathematically,

$$\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla \bullet \vec{v}$$

This says that if an air parcel is experiencing divergence in three dimensions, its density will decrease. Makes sense.

But there's a complication. If the density decreases, then according to the ideal gas law and the first law of thermodynamics, the pressure should decrease too. And if the pressure drops, the air will try to rush back in response to the new pressure gradient, and the density will go back up again. So unless the air parcel is actually ascending or descending, the three-dimensional divergence tends to be very small because changes in density are relatively small. Indeed, except for vertical motion, the only important atmospheric phenomenon that involves rapid changes of density is a sound wave. For most other intents and purposes, the atmosphere can be regarded as *incompressible*, meaning that density essentially doesn't change in response to horizontal motion, and whatever amount of air comes in one part of an arbitrary volume must be going out some other part.



Incompressibility: What goes in must come out, or at least the amounts must be equal. In this example, there is horizontal divergence, but there is an equal amount of air coming in from the top and bottom so that the three-dimensional divergence is zero.

In pressure coordinates, the continuity equation turns out to be much simpler. It is

$$\nabla \bullet \vec{v} = 0$$

Written out completely, this is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

The symbol ω is vertical motion in pressure coordinates. Just as vertical motion in normal Cartesian coordinates is

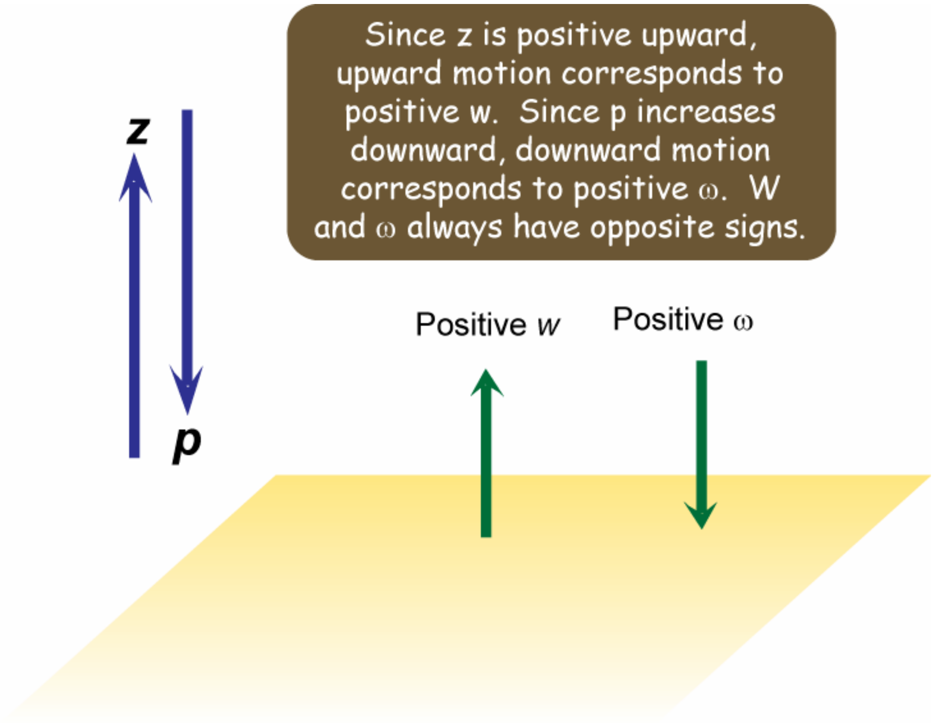
$$w = \frac{Dz}{Dt}$$

which is the rate of change of altitude of an air parcel, the vertical motion in pressure coordinates is

$$\omega = \frac{Dp}{Dt}$$

which is the rate of change of pressure of an air parcel.

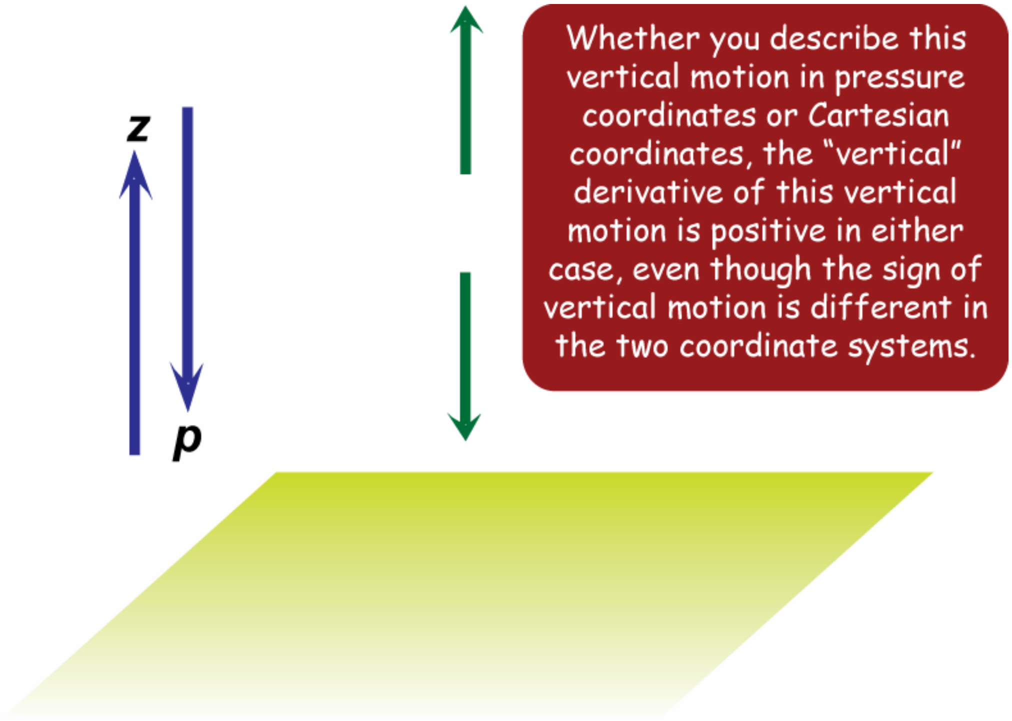
An aside about ω. The vertical motion in pressure coordinates is positive when the air is descending, since the pressure is larger at lower levels. So downward motion is positive in pressure coordinates and negative in height coordinates.



Since z is positive upward, upward motion corresponds to positive w. Since p increases downward, downward motion corresponds to positive ω. W and ω always have opposite signs.

While this is annoying and confusing, the vertical derivative of vertical motion comes out the same sign in both coordinate systems. This means that you can use your normal intuition for the continuity equation no matter which coordinate system you're using.

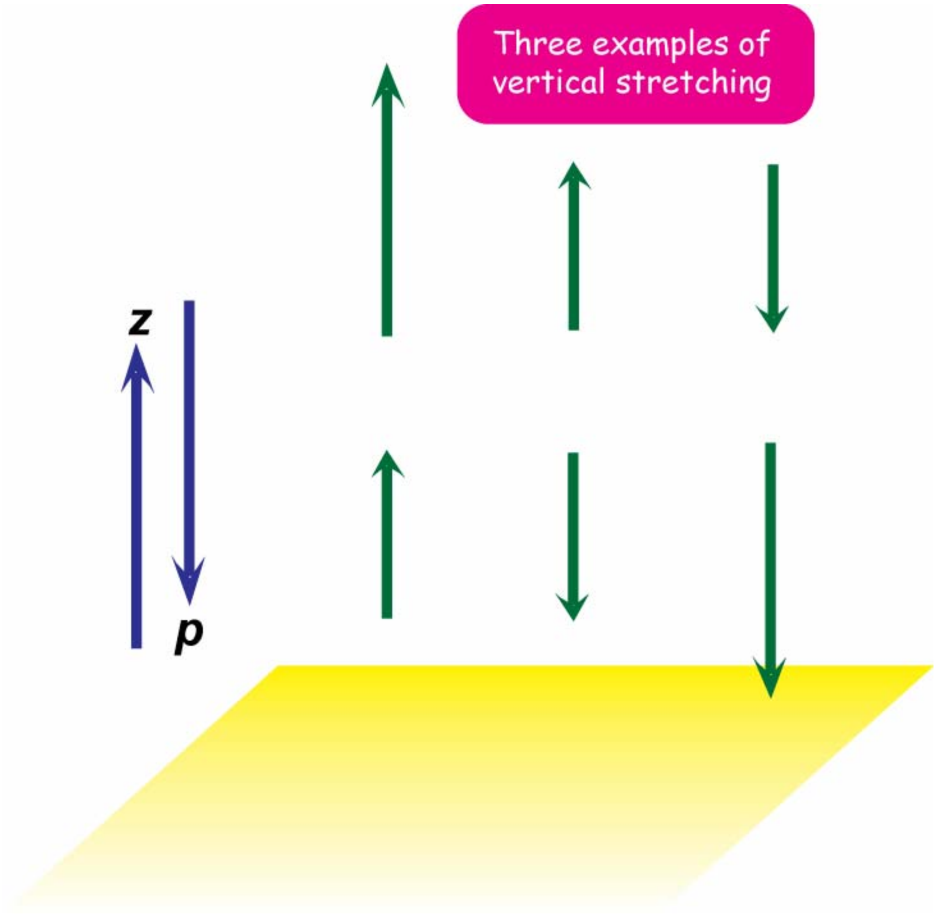
The interpretation of the vertical derivative of vertical motion is "stretching". Imagine a columnal air parcel. For simplicity, let's assume that we have upward motion, but the same argument applies no matter the sign of the vertical motion. If $\partial w/\partial z$ is positive, then the upward motion at the top of the column must be larger than the upward motion at the bottom of the column. It is as though the upward motion is “stretching” the air in the column. Conversely, if $\partial w/\partial z$ is negative, the upward motion would be contracting or “shrinking” the air in the column.



Whether you describe this vertical motion in pressure coordinates or Cartesian coordinates, the “vertical” derivative of this vertical motion is positive in either case, even though the sign of vertical motion is different in the two coordinate systems.

The same thing applies in pressure coordinates. If we have downward motion, ω is positive. If $\partial \omega/\partial p$ is positive, then the downward motion must be strongest at higher pressures, that is, at the base of the column, so again we have stretching of the air column. No matter the vertical motion, and no matter whether we’re talking about $\partial w/\partial z$ or $\partial \omega/\partial p$, if the vertical derivative of vertical motion is positive, the column is stretching.

In pressure coordinates, the three-dimensional divergence is exactly and identically zero everywhere. This is even more “incompressible” than in height coordinates. The only approximation involved is the one that allows the conversion from height to pressure coordinates in the first place: hydrostatic balance. For the rest of this chapter, we’ll stick with the pressure coordinates.



9.5 Vertical Motion and Convergence

According to the continuity equation in pressure coordinates, rearranged slightly,

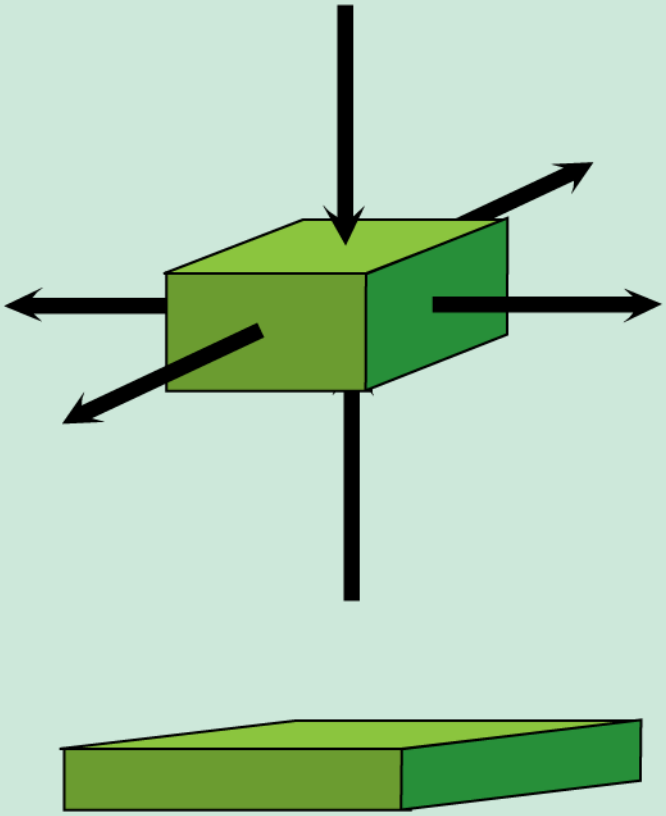
$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

or

$$\frac{\partial \omega}{\partial p} = -\nabla_h \bullet \vec{v}$$

This equation states that stretching of the air column occurs whenever there is horizontal convergence.

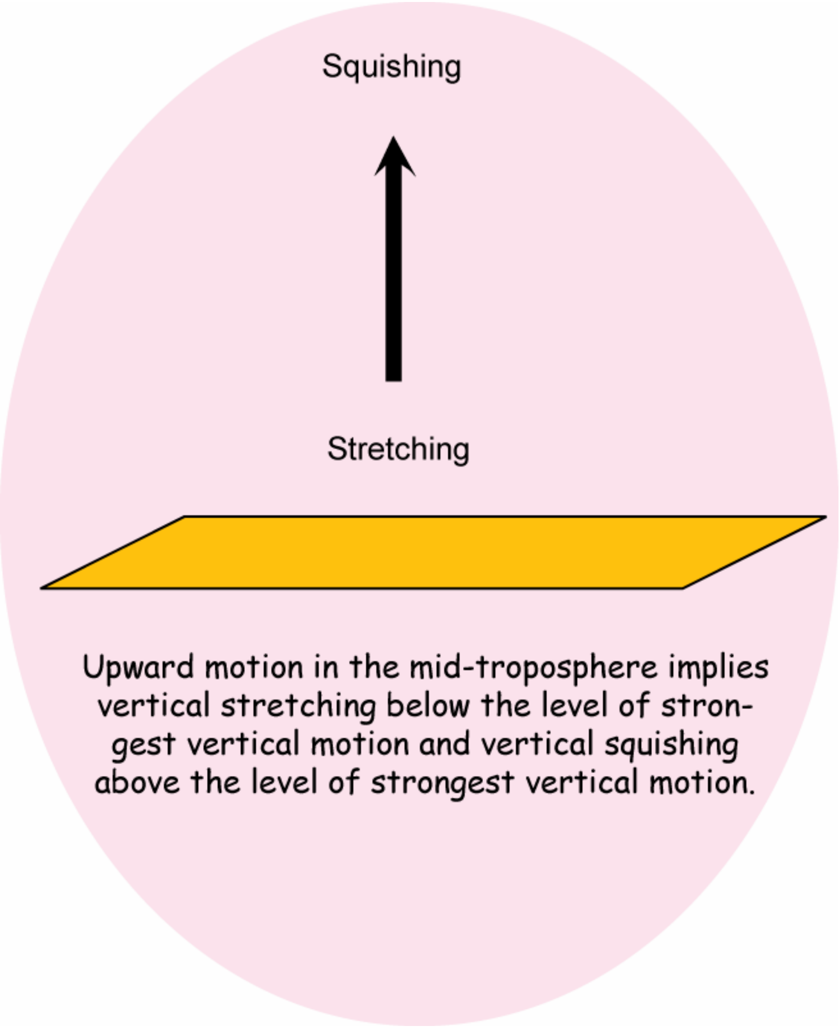
No great leaps of faith are required here. Imagine a cube of jello. If you squeeze the sides of the cube, it will squish outward at the top and bottom. By applying convergence in the horizontal direction, you've stretched it in the vertical direction. A column of air works the same way, even though it doesn't taste as good. Conversely, if horizontal divergence is present, the parcel of jello will be shrunk, or squished, in the vertical direction.



With incompressible flow, horizontal divergence is associated with vertical shrinking, or squishing, of a parcel, be it made of air or of lime jello, turning the parcel on top into the one on the bottom..

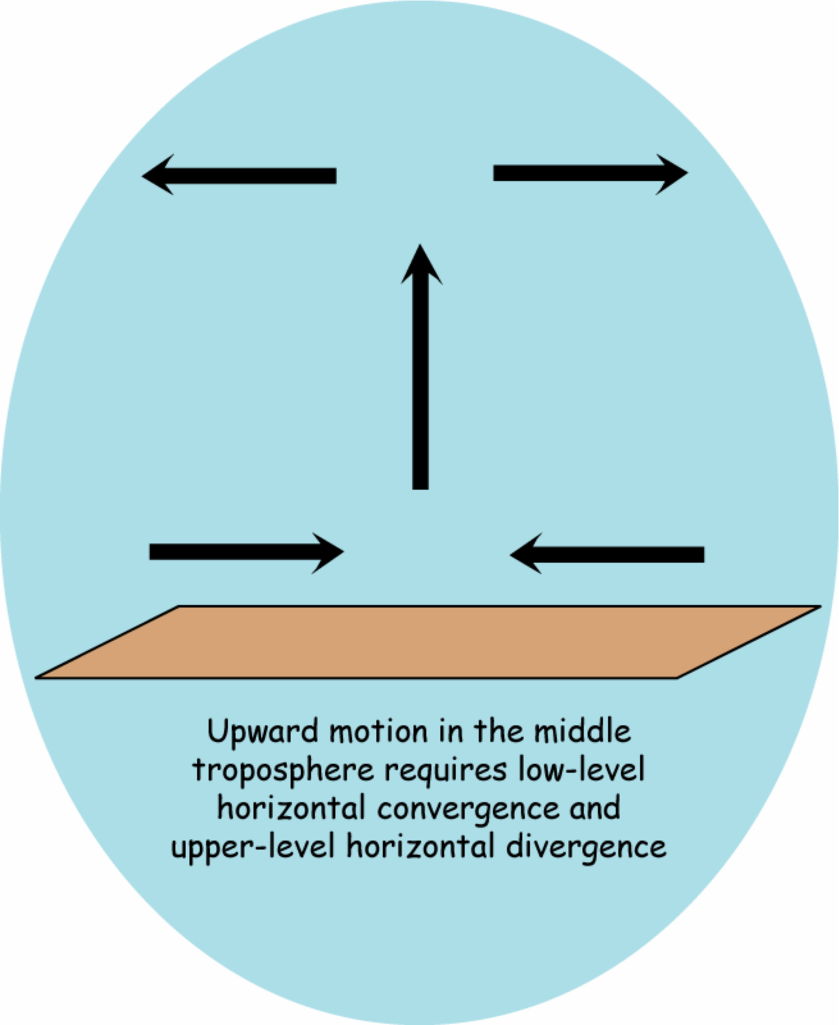
Beware about the multiple meanings of convergence and divergence. When we say convergence and divergence, we could be talking about convergence and divergence in three dimensions. However, since the three-dimensional divergence is nearly (or in pressure coordinates, exactly) zero, there's not much that can be said about it. Instead, when most meteorologists refer to divergence and convergence without specifying, they are referring to the two-dimensional or horizontal divergence and convergence. That's what this book will mean from now on when they talk about divergence and convergence.

Now, anytime you have non-zero vertical motion, you have to have air columns stretching or shrinking, or both. That's because vertical motion tends to be nearly zero at the top of the troposphere, and over a flat ground (or the ocean) it must be zero too. So a large upward motion somewhere in between those two levels means that air columns must be experiencing stretching at low levels and shrinking at high levels within the troposphere.



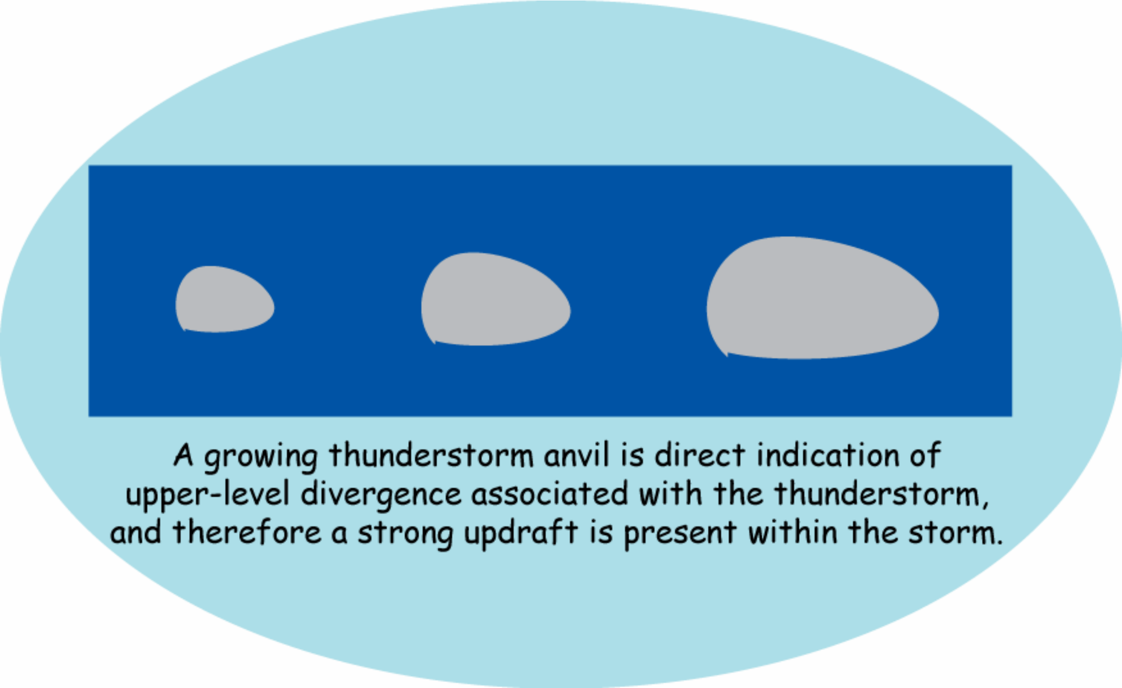
So put two and two together. If you have upward motion in the mid-troposphere over a flat surface, you have stretching below that level and shrinking above. According to the continuity equation, if you have stretching you must have horizontal convergence, and if you have shrinking you must have horizontal divergence. Therefore, mid-tropospheric upward motion implies low-level convergence and upper-level divergence.

Notice that this is a diagnostic relationship. It is just as correct (and meaningless) to say that convergence or divergence cause vertical motion as it is to say that vertical motion causes convergence and divergence. For cause and effect, you must identify which of these three things (vertical motion, convergence, and divergence) is being directly caused by something else.



At the surface near a low, we can do that. Surface friction causes low-level air to flow inward toward a surface low. Thus, at a surface low, there is low-level convergence caused by the surface friction. This, in turn, causes mid-tropospheric vertical motion and upper-level divergence, assuming the low is large enough to affect the entire troposphere. This isn't the main reason there's upward motion near low-pressure centers, but it is a contributing factor.

Sometimes, upper-level divergence is also easy to observe. Consider a satellite loop that features an outbreak of thunderstorm activity. The anvils spread out impressively during the first few hours of development. The outward spread of the anvils is a visual manifestation of the horizontal divergence taking place aloft. And, according to the continuity equation, the air columns must be shrinking and flattening, presumably because of strong upward motion below anvil level. That updraft, in turn, implies strong convergence at low levels feeding into the thunderstorms.



A growing thunderstorm anvil is direct indication of upper-level divergence associated with the thunderstorm, and therefore a strong updraft is present within the storm.

9.6 Computing Vertical Motion from Divergence

Consider again the equation

$$\frac{\partial \omega}{\partial p} = -\nabla_h \bullet \vec{v}$$

If we integrate both sides from some bottom pressure p_o to an arbitrary pressure p_1, we get

$$\omega_1 = \omega_0 - \int_{p_0}^{p_1} (\nabla \bullet \vec{v}) dp$$

According to this equation, the average divergence between the two pressure levels, multiplied by the pressure interval itself, gives you the difference in vertical motion (in pressure coordinates) between the two levels.

One special case for this equation is if the lower pressure level is at the ground. If the ground is flat, air parcels at the ground won’t have their pressure change much, and the first term on the right hand side of the equation is approximately zero. Thus, the low-level average divergence gives you the vertical motion at the top of that layer.

Let's work that example through. Suppose the average divergence in the lowest 200 mb of the atmosphere is -5 x 10^{-5} s^{-1}. The negative value means convergence, so we expect upward motion. The integrand is negative, but $p_1 – p_0$ is -200 mb, so the integral is the product of two negative numbers: 1 x 10^{-2} mb s^{-1}. Assuming the change in pressure at the ground (ω_0) is zero, ω_1 is 0 mb s^{-1} – 1 x 10^{-2} mb s^{-1}, or -1 x 10^{-2} mb s^{-1}. In physical terms, an air parcel at roughly 800 mb (200 mb above the ground) will be rising at the rate of 1 mb per 100 s.

Such a calculation, in some circumstances, is the best way of determining the vertical motion. It is used in radar analysis and in computing vertical motion from rawinsonde measurements. The difficulty with such a calculation is that small errors in the wind measurements can lead to large errors in the computed divergence and therefore large errors in the computed vertical motion. Thus, this approach is rarely applied in a quantitative fashion to winds determined from rawinsonde observations.

We bring it up here to show that, in this context, the integrated divergence is one of the two elements of vertical motion at a particular level. The other element is the vertical motion at the ground. This latter bit of vertical motion is the focus of the next subsection.

9.7 Vertical Motion at the Ground

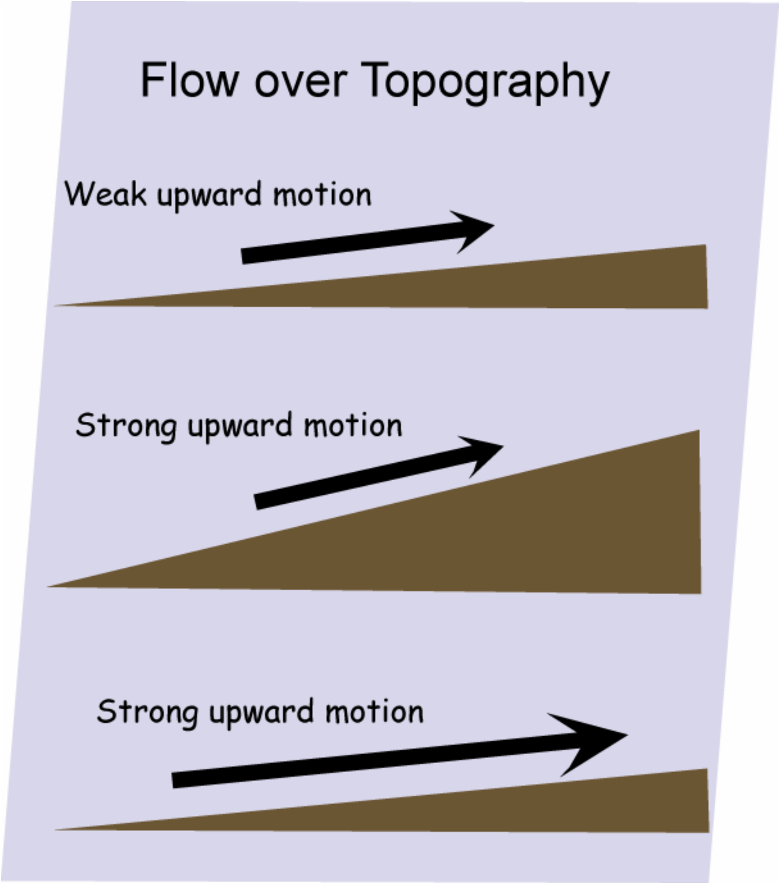
Up to this point, we have considered only cases in which the ground is flat. In height coordinates, this is enough to ensure that the vertical motion at the ground is zero. In pressure coordinates, you can get a slight nonzero vertical motion just from the fact that the air pressure at the ground is not constant. A changing air pressure implies a nonzero ω. Even so, this will be small compared to the size of ω in the free atmosphere.

The situation changes if the ground is sloped. A slope means that even "horizontal" wind can cause upward motion. The horizontal wind must have a component up or down the slope; if the wind is just blowing parallel to the slope it will be blowing neither up nor down. The magnitude of the ascent caused by wind over sloping terrain depends on the strength of the component up or down the slope and the steepness of the slope itself. Indeed, this is another case of the dot product: if H is the height of the terrain, then

$$w = \vec{v} \bullet \nabla_h H$$

Study this equation until you are convinced that it makes sense.

In areas such as the western United States, wintertime precipitation is strongly affected by the topography. Where the air is flowing up the mountains during a winter storm, vertical motion is that much stronger and the precipitation is copious. On the lee side, the downslope flow induces much weaker ascent aloft or maybe even descent throughout the troposphere, resulting in little or no precipitation on the lee side.



9.8 Advection and the Total Derivative

In a moment, we'll discuss the role of vertical motion in temperature changes, but first, an aside about the total derivative.

Advection is actually ubiquitous in the equations that describe the behavior of the atmosphere. Most such equations describe processes that change the characteristics of an air parcel. However, for forecasting and understanding purposes, it is important to know how and why weather elements at a particular location are changing, or why the weather patterns are evolving the way they do. So while the total derivative is the rate of change following an air parcel, we usually need to know the rate of change at a particular location. In mathematical terms, with *A* being an arbitrary meteorological variable, given

$$\frac{DA}{Dt}$$

we need to know

$$\frac{\partial A}{\partial t}$$

The two are related as follows:

$$\frac{\partial A}{\partial t} = -\vec{v} \bullet \nabla A + \frac{DA}{Dt}$$

Thus, if you know everything affecting an air parcel, the only additional term affecting the evolution *at a particular location* is advection.

Consider the equation governing water vapor. Water vapor is a substance, the gaseous phase of water. In the atmosphere, it is a stable, non-reactive molecule. So, the only processes that change the amount of water vapor in an air parcel are those that involve phase changes of water: condensation and evaporation. (We'll lump deposition and sublimation in with condensation and evaporation for the sake of simplicity.) If we designate the rate of evaporation as E and the rate of condensation as C, the mathematical form of the above statement for specific humidity *q* is

$$\frac{Dq}{Dt} = E - C$$

That's fine, but what happens to water vapor at a particular location? Simple. Just add advection:

$$\frac{\partial q}{\partial t} = -\vec{v} \bullet \nabla q + E - C$$

Similar sorts of equations, some possibly involving chemical reactions, apply to all the other constituents of the atmosphere.

9.9 The Thermodynamic Equation, Revisited

Let’s consider another quantity, temperature. Temperature changes, following an air parcel, because of heat energy added or removed from the parcel (designate it Q), but also because of adiabatic expansion and contraction. In height coordinates, a rising or descending air parcel cools or warms at the dry adiabatic lapse rate, 10K/km. To convert this to a rate in time, one would multiply it by the speed at which the air is ascending or descending. So the equation governing temperature (the thermodynamic equation) would look like this, where Γ_d is the dry adiabatic lapse rate, defined as a positive number (the rate of cooling with height):

$$\frac{DT}{Dt} = -w\Gamma_d + \frac{Q}{c_p}$$

To convert this into an equation governing the change in temperature at a particular location, simply add advection:

$$\frac{\partial T}{\partial t} = -\vec{v} \bullet \nabla T - w\Gamma_d + \frac{Q}{c_p}$$

So the three processes that change temperature at a particular location are diabatic heating and cooling (the third term on the right), adiabatic heating and cooling (the second term on the right), and advection (the first term on the right).

The diabatic heating can come about through a variety of processes: heating or cooling due to absorption or emission of radiation (usually a small effect on daily time scales), heating or cooling due to condensation or evaporation, and heating or cooling due to contact with warmer or colder material, such as the Earth’s surface. For weather forecasting, if you’re away from the boundary layer and the air is dry, the diabatic heating term is small enough to ignore.

The adiabatic heating and cooling term is rather interesting. Notice that it is proportional to the magnitude of the vertical component of the wind. But it is not the only term proportional to the vertical component of the wind – if we expand the advection into horizontal and vertical components we get

$$\frac{\partial T}{\partial t} = -\vec{v} \bullet \nabla_h T - w\frac{\partial T}{\partial z} - w\Gamma_d + \frac{Q}{c_p}$$

This is not especially useful. If there’s vertical motion going on, we want to know its net effect on temperature, not two separate effects that we then have to add together. So combining the two terms involving *w*, we get

$$\frac{\partial T}{\partial t} = -\vec{v} \bullet \nabla_h T - w\left(\frac{\partial T}{\partial z} + \Gamma_d\right) + \frac{Q}{c_p}$$

This says that the change in temperature at a particular point due to vertical motion is proportional to the difference between the actual lapse rate and the adiabatic lapse rate. (Recall that the lapse rate is the negative of the vertical change of temperature.) In other words, in a neutrally-stratified atmosphere, vertical motion would produce no change in temperature at a particular level. In contrast, in a strongly stratified atmospheric layer, the difference between the actual lapse rate and the dry adiabatic lapse rate is large and the effect of vertical motion on temperature is large too. In a temperature inversion, the temperature change at a particular level is even more rapid.

It is important to understand and distinguish between these two processes. As long as there’s no condensation or evaporation going on, an ascending or descending air parcel will always cool or warm at the dry adiabatic lapse rate. The change in temperature at a particular level is, in general, different, and is proportional to the difference between the dry adiabatic lapse rate and the actual lapse rate.

In Chapter 5, these principles were worked out graphically on a sounding diagram. Back then, to determine how the temperature of an air parcel changed due to vertical motion, we raised or lowered the parcel along the dry adiabat. The first equation in this section corresponds to this process, assuming no diabatic heating.

When considering the change in temperature *at a particular level*, what matters is not just the change in temperature of a parcel lifted, but the difference between that lifted temperature and the original temperature at the level in question. Thus, vertical variation of the actual temperature matters for local changes in temperature.

Following an air parcel, upward motion always causes cooling. At a particular level, rather than following an air parcel, it is possible for upward motion to actually cause warming. This happens if the actual lapse rate is even steeper than the adiabatic lapse rate, in other words, when the atmosphere is unstable. The most common situation is one in which a given layer in the troposphere is weakly stable or neutral. In that case, upward motion causes little or no cooling in that layer, respectively.

The potential temperature of an air parcel does not change due to adiabatic expansion or compression, because it’s defined with respect to a reference pressure. So the equation governing the potential temperature has one less term in it. The diabatic heating term gets modified slightly to convert between temperature and potential temperature, but otherwise the equation governing the evolution of potential temperature is just as you might imagine it:

$$\frac{\partial \theta}{\partial t} = -\vec{v} \bullet \nabla_h \theta - w\frac{\partial \theta}{\partial z} + \frac{Q}{c_p}$$

Questions

1. In a cubical volume of air, the wind is blowing in from the west at 10 m s^{-1} and blowing out from the east at 15 m s^{-1}. State two possible combinations of wind values on the other four sides of the cube that are consistent with the three-dimensional divergence being zero.

2. Suppose the horizontal wind is blowing directly outward in all directions from a particular point. Suppose too that the speed of this wind increases with distance from that point at the rate of 1 m s^{-1} per km. Using a circle with its center at the origin, demonstrate that the divergence theorem is correct.

3. Suppose the *u* component of the horizontal wind increases from west to east at the rate of 5 m s^{-1} per 100 km. If the horizontal divergence is exactly zero everywhere, how does the *v* component of the horizontal wind vary? Sketch a wind field (over a 300 km by 300 km square) consistent with these wind component variations.

4. Suppose the air is moving downward at the rate of 0.1 mb s^{-1} at 700 mb, 0.04 mb s^{-1} at 800 mb, 0.01 mb s^{-1} at 900 mb, and 0 at 1000 mb. Compute the convergence or divergence within these layers.

5. Suppose the surface wind is blowing from the southwest at 12 m s^{-1} and the ground slopes upward from west to east at the rate of 100 ft per mile. Compute the vertical component of motion. Express the answer in units of cm s^{-1}.

6. Suppose the temperature at a particular location is decreasing at the rate of 1 C per hour. You check the weather map and notice that the horizontal temperature gradient is oriented toward the south at a magnitude of 1 C per 70 km. (a) If the wind is blowing from the north, how strong would it have to be to account for the observed temperature change? (b) If the wind is blowing from the east, is the vertical motion more likely to be upward or downward? (c) If the temperature decreases upward at the rate of 3 C km^{-1}, how strong is the vertical motion?