

# Weather Observation and Analysis

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## Course Notes

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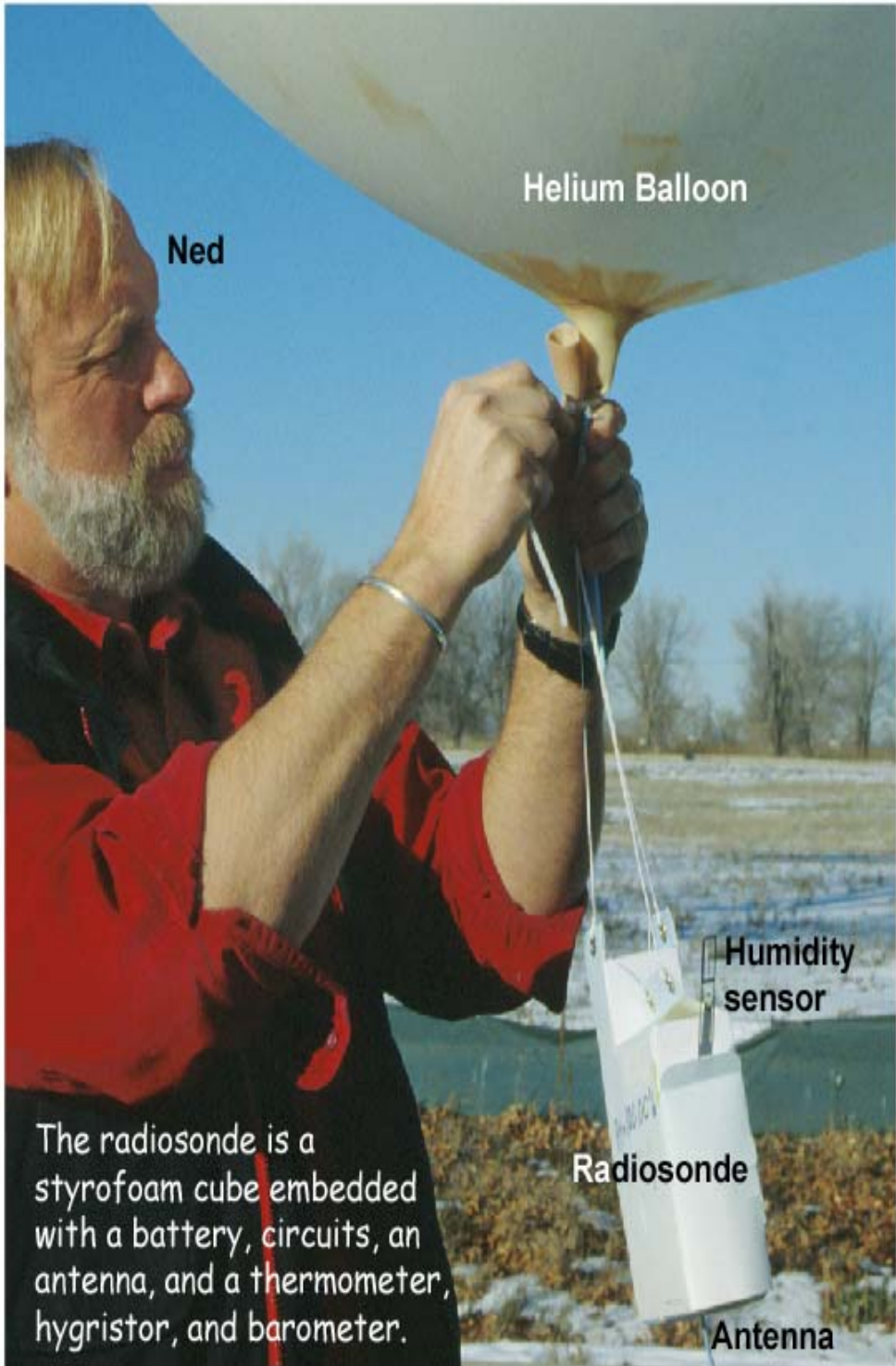
## Chapter 5. SOUNDINGS

### 5.1 Sounding Observations

While airplanes are helpful, the primary tool for probing the detailed vertical atmosphere has long been the rawinsonde. After World War II, the coordinated launch of soundings became a worldwide effort. The timing of rawinsonde launches, at 0000 UTC and 1200 UTC, is the main reason for twelve hours being a common interval for forecast model runs. It is at those times that the greatest amount of upper air data are available and the most accurate analyses are likely.

There exist four basic types of sounding observations. Radiosondes are instrument packages that include sensors for temperature, humidity, and pressure, all attached to a balloon filled with a lighter-than-air gas such as helium. Because pressure decreases upward at a rate that depends on the temperature and humidity, the combination of sensors makes it possible to determine the altitude of the radiosonde and construct vertical profiles of temperature and humidity with respect to both height and pressure.

Pibals, short for ‘pilot balloons’, have no instrument package at all. Their purpose is to provide a vertical profile of the wind speed and direction. To accomplish this, their position is tracked from a ground station using a direction-finding system known as a theodolite. The direction (relative to north) of the balloon and the elevation angle (relative to horizontal) provide two of the three independent bits of information needed to pin down the balloon’s location. The third bit is obtained by ensuring that the balloon is inflated precisely so that it will ascend at a





NWS Forecaster Troy Lindquist using a theodolite to track a pibal and determine the winds in the vicinity of a forest fire near Grand Junction, Colorado. The horizontal and vertical wheels display the azimuth and elevation angles of the balloon.

known speed. This is done by attaching weights to the bottom of the balloon and then inflating the balloon until the weights just barely lift off the ground. By keeping track of the elapsed time after release, the height of the balloon can be computed. Finally, wind is determined from the change in horizontal position of the balloon from one time to the next as it is carried by the wind.

Rawinsondes combine the thermodynamic information of radiosondes with the wind information of pibals. The height of the balloon need not be estimated from elapsed time because it can be computed from the measured vertical profiles of temperature, humidity, and pressure. Modern rawinsondes are equipped with GPS transmitters that detect and transmit the balloon's location using GPS navigation, making tracking with a ground station unnecessary. As with pibals, the wind is determined from the drift of the balloon over time.

If you're wondering why they don't just measure wind speed and direction on the rawinsonde with a standard anemometer and wind vane, there's one problem with that. Since the balloon is drifting with the wind,

there's no significant air motion relative to the balloon. Both balloon and air are moving at the same speed and direction (except that the balloon is also going up), so an anemometer on a rawinsonde would always report calm winds, even when the balloon is drifting along at 100 miles per hour.

The final class of soundings are merely upside-down radiosondes and rawinsondes. Called 'dropsondes' or 'dropwindsondes', they are dropped from an airplane and drift down with the help of a parachute or appropriate aerodynamic design. A dropwindsonde measures the same things on the way down as a rawinsonde measures on the way up. Since dropsondes eventually hit the ground, they must be lightweight and made of soft material, mostly Styrofoam. Radiosondes suffer the same fate and come under similar design constraints, since eventually the ascending balloon gets so big that it bursts.

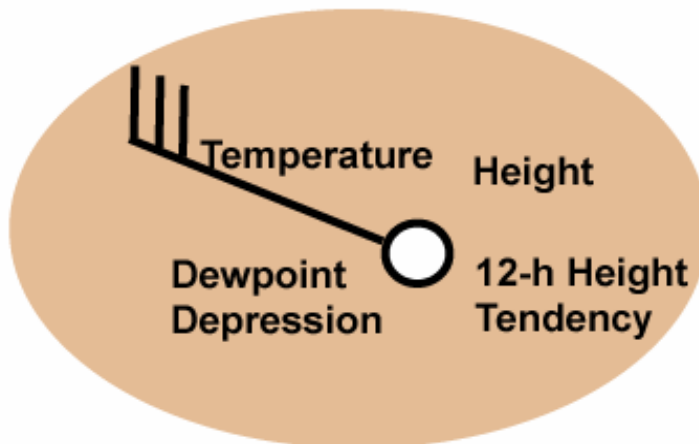
## 5.2 Upper Air Maps

One common way of displaying upper air data is on a horizontal or quasi-horizontal map. I add the "quasi-" part because most upper air maps are not on a flat surface some fixed distance above sea level. Most commonly, the maps are plotted with all the data corresponding to a single value of pressure.

To facilitate the plotting of upper air maps at pressure levels, data from certain pressure levels are required to be transmitted as part of every radiosonde report. These particular levels are called 'mandatory levels' for that reason. The mandatory pressure levels up to and including 100 mb are: 1000 mb, 925 mb, 850 mb, 700 mb, 500 mb, 400 mb, 300 mb, 250 mb, 200 mb, 150 mb, and 100 mb. Of these, 850 mb, 700 mb, 500 mb, 300 mb, 250 mb, and 200 mb are most commonly used for maps. There are three other mandatory levels that don't correspond to particular pressures: the ground surface, the level of maximum winds (jet level), and the tropopause.

Upper air maps have a fixed plotting convention similar to surface maps, except that there are much fewer weather elements plotted. The convention for an upper-air constant pressure map is as follows:

- The station location is represented by a symbol that identifies the type of observation. The most common types are radiosonde observations (circles), aircraft observations (squares), and satellite cloud wind vectors (stars).



- The wind barb is drawn ending at the station location using the conventions described in Section 3.2.

- The temperature, in whole degrees Celsius, is plotted to the northwest of the station location, as with surface maps.

- The height of the pressure level is plotted in meters to the northeast of the station location, where the pressure goes on surface maps. Only the last three digits are shown. At pressures greater than 500 mb, the height is given in meters, while at pressures of 500 mb or less, the height is given in decameters (that is, tens of meters, abbreviated dam). Thus, for example, a 500 mb height of 5462 m is written as 546. It is useful to memorize the typical heights of the most common pressure surfaces so that you can tell what digits, if any, have been dropped: 850 mb ~ 1500 m; 700 mb ~ 3000 m; 500 mb ~ 5400 m; 300 mb ~ 9000 m; 250 mb ~ 10000 m; 200 mb ~ 11000 m.

- The dewpoint depression is plotted to the southwest of the station location, where the dewpoint normally goes in surface maps. The dewpoint depression is defined as the temperature minus the dewpoint. When the relative humidity is high, this will be a small number. Dewpoint depression is plotted rather than dewpoint itself because usually it is the depression that matters: very small depressions indicate cloudy air and moderately small depressions indicate that clouds could easily form with just a little ascent. When the dewpoint depression is less than 5 C, the station circle is filled in.

- At radiosonde sites, where observations are taken at regular intervals, the height tendency is plotted to the southeast of the station location, where the pressure tendency belongs on surface maps. Instead of

a symbol indicating the type of height tendency, only a plus or minus is used. The height change over the past 12 hours is given in meters.

### 5.3 Sounding Diagrams

Sounding diagrams are the conventional way of displaying the vertical profiles of thermodynamic quantities recorded by a radiosonde. In addition to their core role of depicting the vertical structure of the atmosphere, they have various forecasting applications: forecasting precipitation type, forecasting boundary-layer evolution (clouds and temperatures), and forecasting the occurrence and type of convection.

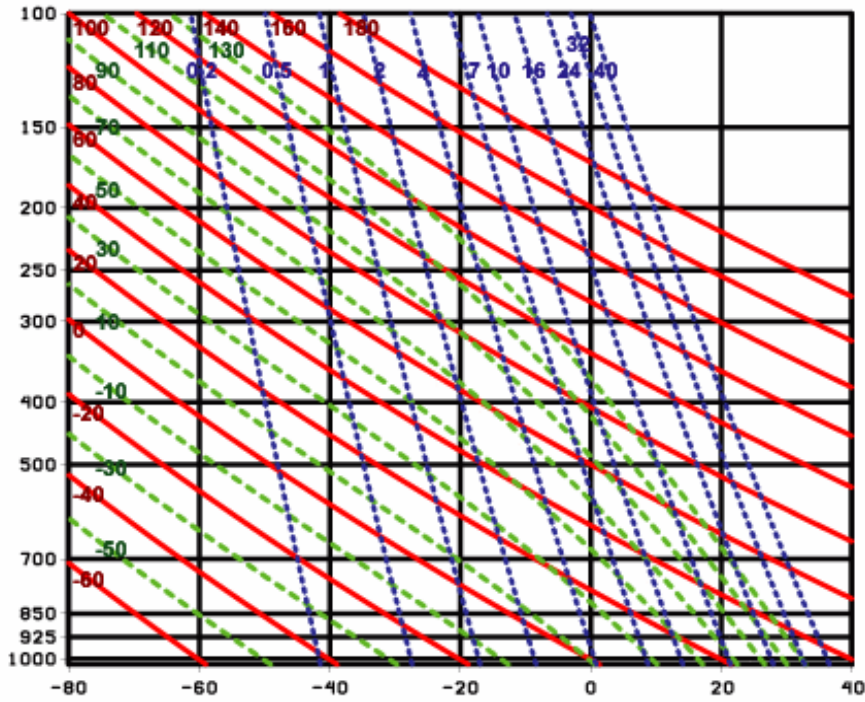
A sounding diagram is a type of *thermodynamic chart*. Such charts are widely used in thermodynamics for depicting the states (and transitions) of thermodynamic systems. For a well-mixed system, that state can be specified by pressure and temperature. And since pressure decreases upward, a plot of the thermodynamic states of the air as encountered by the ascending radiosonde serves as a graph of the vertical structure of the atmosphere.

Sounding diagrams come in many forms. The simplest possible sounding charts use pressure as a vertical coordinate in either logarithmic or linear fashion. Among the types of charts in common use are the Stüve diagram, the emagram, the tephigram, and the skew-T log-p diagram.

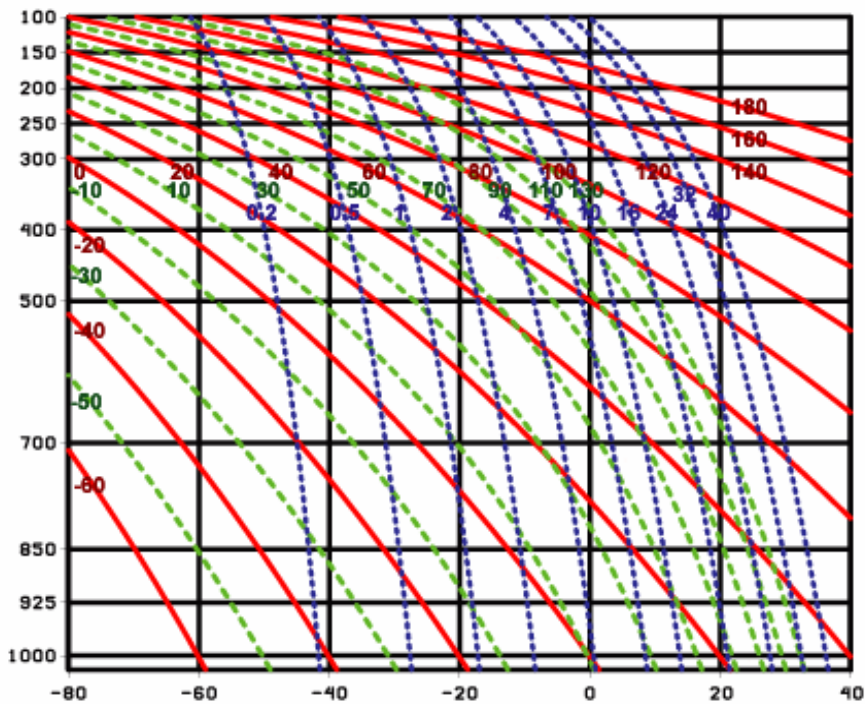
The following pages give examples of four sounding diagrams: logarithmic, linear, Stüve, and Skew-T log-p (or Skew-T for short). These charts come in various shapes, and may have a few or many lines drawn on them, but they all share the same basic characteristics.

The vertical, or nearly vertical, axis is pressure. This can be interpreted as a surrogate for height, so a sounding diagram can be interpreted as an ordinary graph flipped sideways, with (vertical) position given by the vertical axis rather than the horizontal axis. The other required variable is temperature. The temperature labels are usually found along the bottom of the chart, but the lines of constant temperature tend not to be straight up-and-down as they would be for an ordinary graph. Instead, the temperature lines tend to slope upward toward the right (hence the name skew-T). Other sounding diagrams use various angles and curved (or straight) lines for temperature and pressure in order to maximize the ease of some particular diagnostic analysis or interpretation.

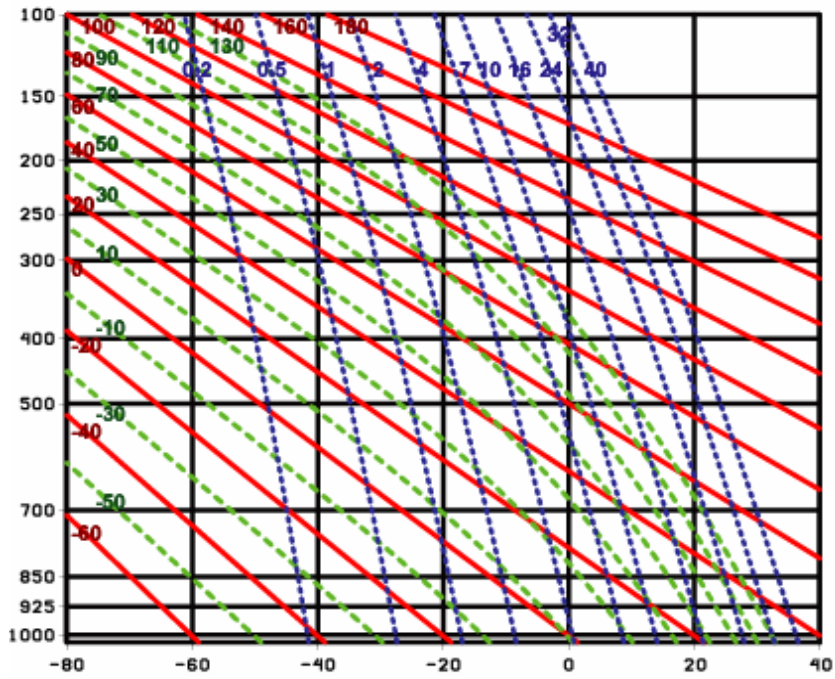
Now suppose you have a data point from a sounding. The pressure at that data point is 778 mb, and the temperature at that data point is 5 C. To find that point on a sounding diagram, first find the 778 mb level. There won't be a line labeled '778', but you will find an 800 mb line and a



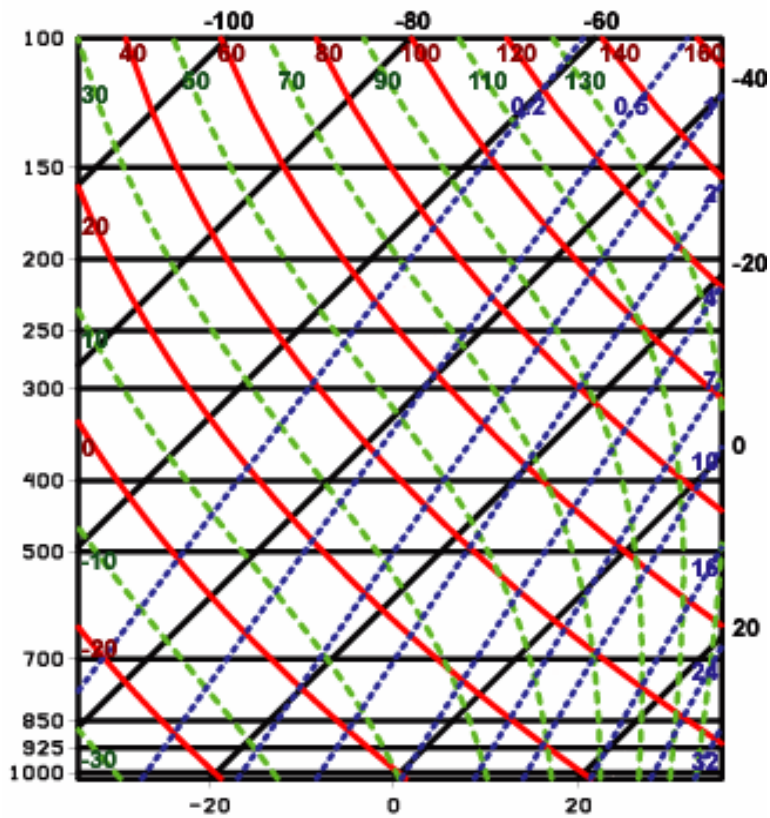
A log-pressure sounding diagram.



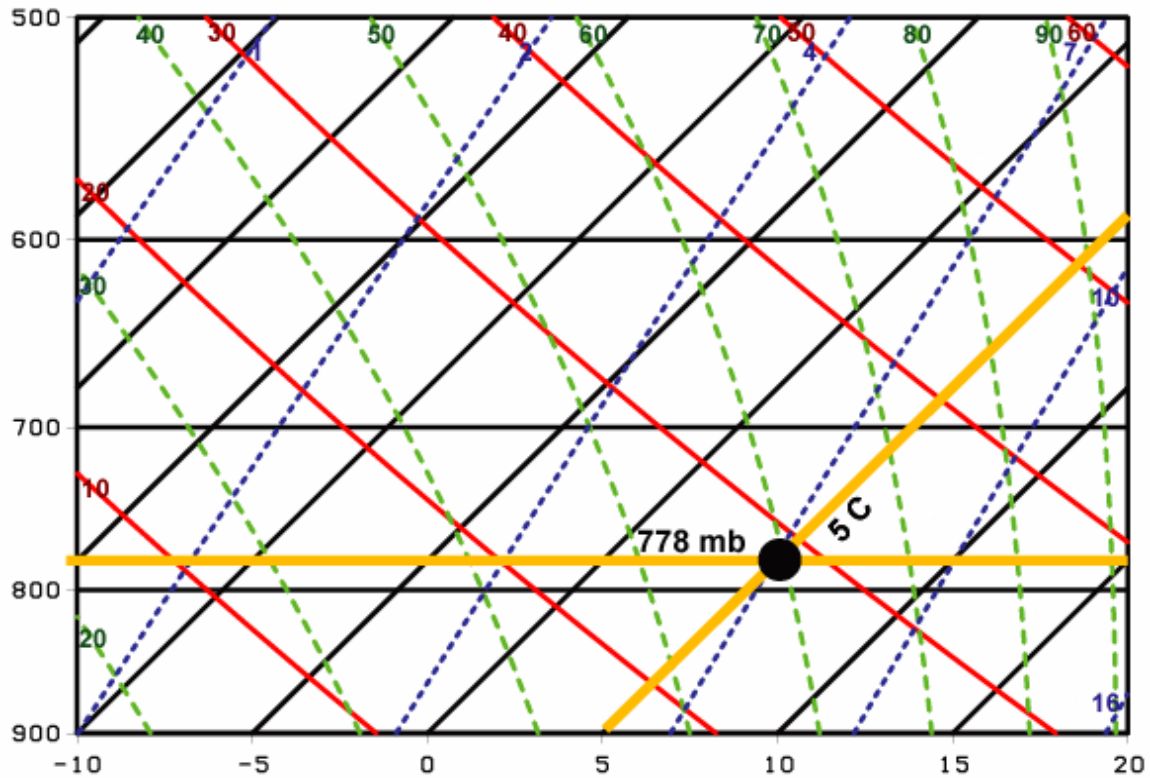
A linear pressure sounding diagram.



A Skew-T log-p diagram.



A Skew-T log-p diagram.

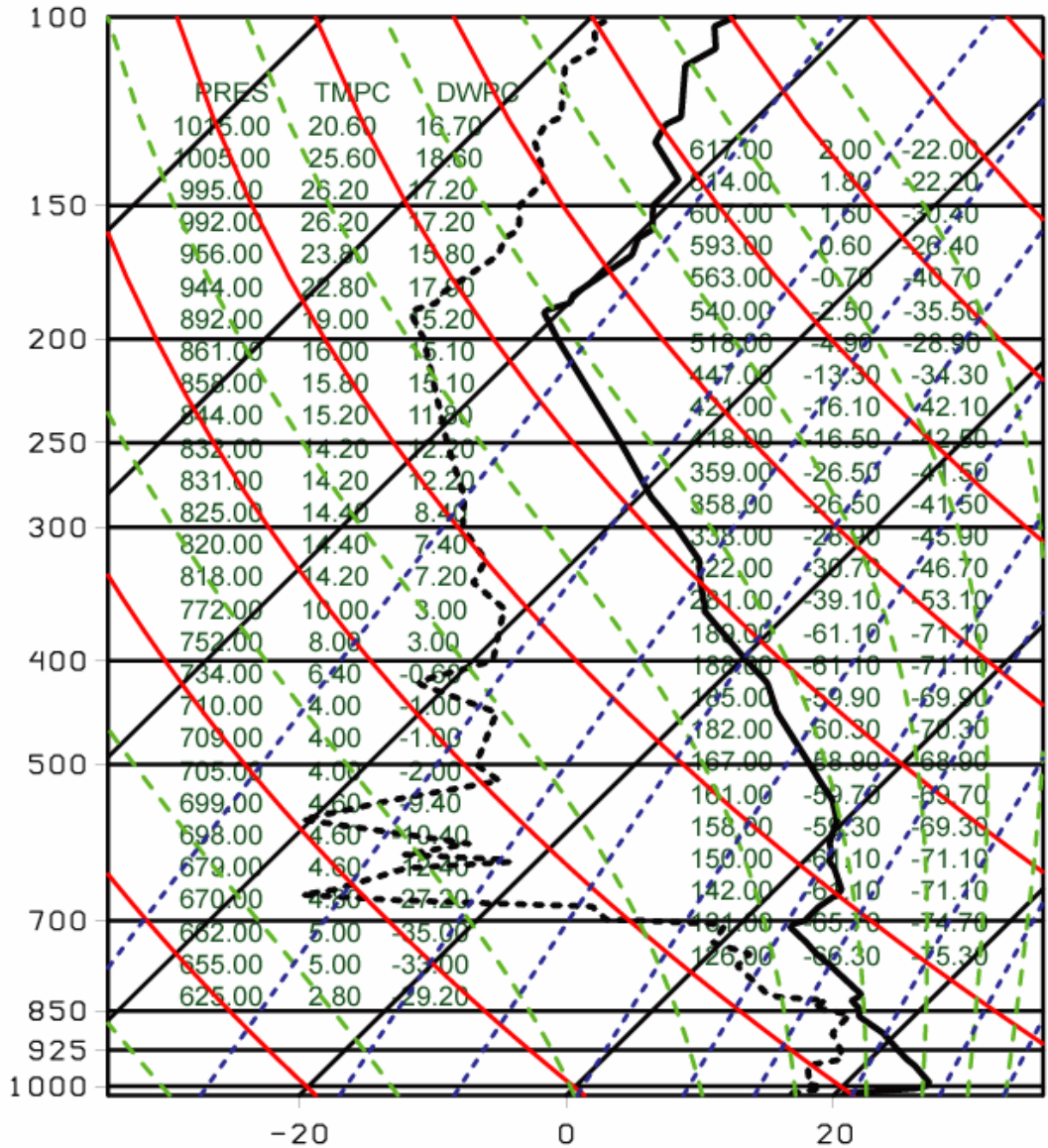


The dot is at the location on the sounding diagram corresponding to 778 mb and 5 C.

750 mb line at least (778 mb is about midway between the two), and if you're lucky a 780 mb line and a 770 mb line (the 778 mb level is closer to 780 mb than 770 mb). Having found the pressure level of the data point, find the two temperature lines that bound the temperature observation. Again, if you're lucky, there will be a 5 C line. If so, just find where the 5 C line crosses the 778 mb level. That is the only point on the sounding diagram with both a pressure of 778 mb and a temperature of 5 C. Draw a dot there, that's your data point. If you're unlucky and there're only a 0 C and 10 C line, you should know that 5 C is halfway between them, and it should still be fairly easy to find where that temperature value belongs at 778 mb.

In its simplest form, one may record the value of temperature at regularly-spaced pressure levels as a weather balloon ascends, plot the (p,T) points on a sounding diagram, and connect the dots to obtain a sounding. Indeed, enough data points are conventionally transmitted from a sounding so that 'connecting the dots' with straight lines gives a vertical profile of temperature that is within one degree of the observed value at all levels. The same procedure may be followed to reconstruct the dew point profile. Winds are ordinarily plotted along the right-hand margin of the

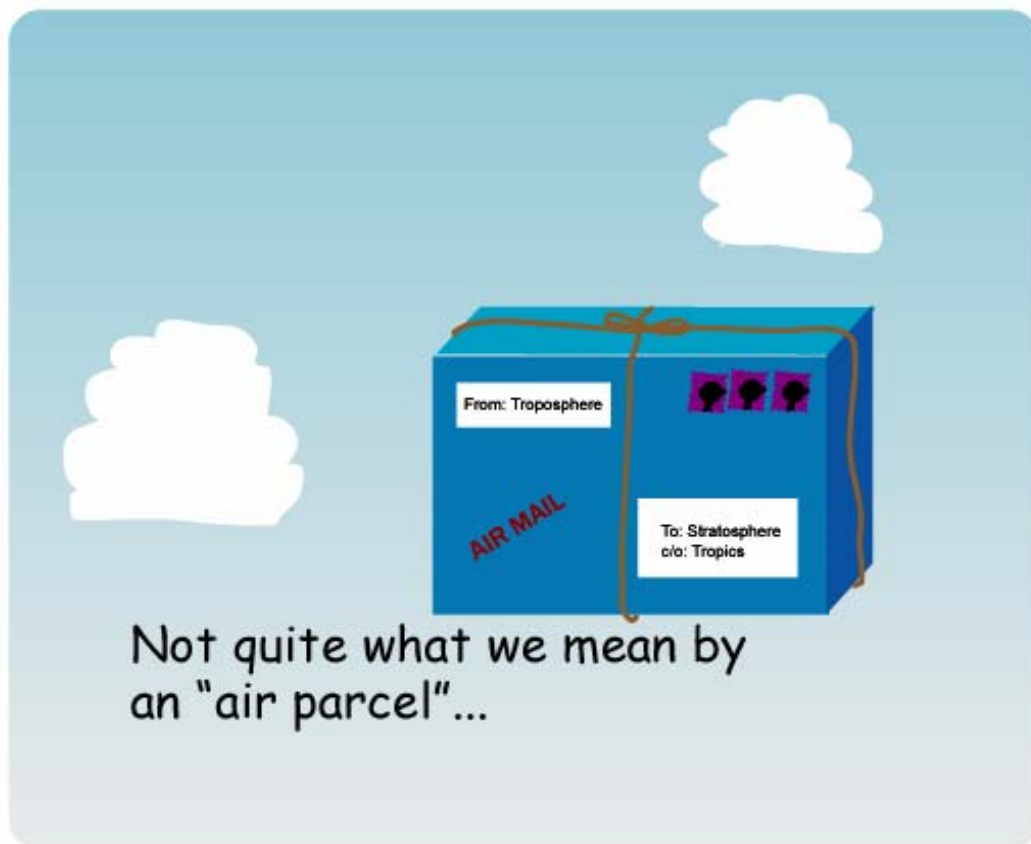
sounding diagram in conventional wind barb format at appropriate pressure levels, with north (for wind direction plotting purposes) being at the top of the page.



Notice the 1:1 correspondence between each element of data and the plotted temperature (black solid) and dewpoint (black dashed) on the sounding diagram. The skewed temperature lines are plotted 20 C apart; use that fact to infer the values of the unlabeled temperature lines.

The rest of this chapter is going to make heavy use of something called an “air parcel”. This is a crude but useful attempt to take the laws of physics as they apply to solid objects and use them on the air. To do this, we have to take some mass of air and treat it as having uniform thermodynamic characteristics. Furthermore, we have to imagine this mass of air staying together, as though it was enclosed inside a balloon.

An air parcel has an indeterminate size and shape. For most of what we’ll use the concept of an air parcel for, the diameter of the air parcel can be visualized as about 100 m.



An air parcel has the following key, imaginary property: all the air within the air parcel moves together, and furthermore all the air in its path cooperates with its motion by getting out of the way. Thus, an idealized air parcel does not experience wind resistance but instead is floating freely in the air.

## 5.4 Thermodynamic Quantities from Soundings

Other lines besides temperature and pressure are usually present on a sounding diagram as well. These lines (some straight, some curved) include lines of constant potential temperature (or ‘dry adiabats’), lines of constant saturation equivalent potential temperature (or ‘moist adiabats’), and lines of constant saturation mixing ratio (or ‘mixing ratio lines’; maybe we should hold a line naming contest). Before discussing what these lines look like on a sounding diagram, here are some brief definitions.

*Potential temperature:* The temperature a parcel of air would have if it were brought to some particular standard pressure or elevation without exchanging heat with its environment and without phase changes of water. Potential temperature is a function of a parcel’s temperature and pressure, as given by Poisson’s Equation:  $\theta = T (p_o/p)^{R/c_p}$ , where  $p_o$  is the reference pressure,  $R$  is the gas constant of dry air (287 J/kg) and  $c_p$  is the specific heat capacity of dry air at constant pressure (1004 J/kg). The ratio  $R/c_p$  is 2/7. Unless otherwise specified, the reference pressure is 1000 mb.

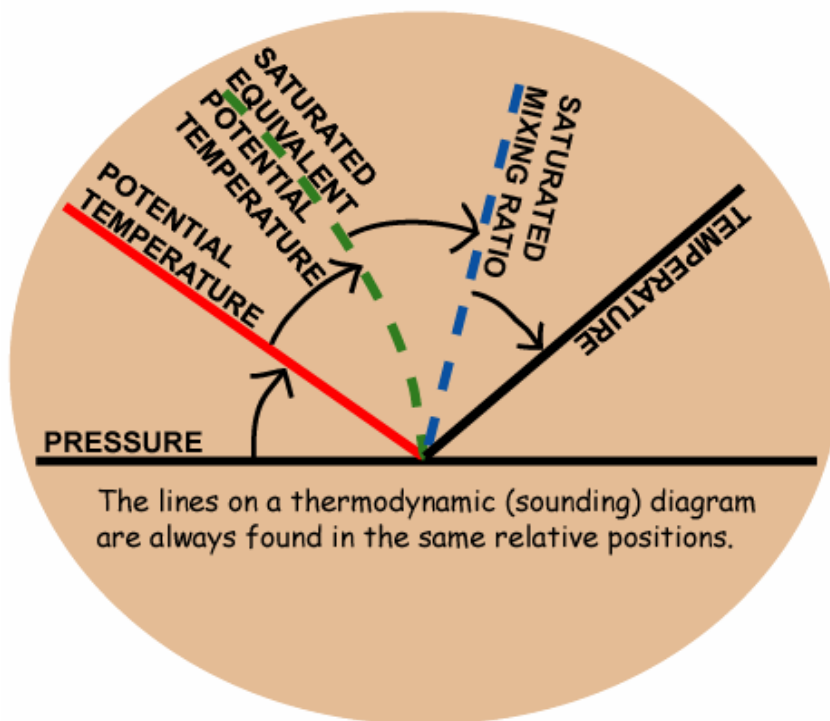
*Saturation Mixing Ratio:* The mixing ratio an air parcel at a given temperature would have if it were saturated, that is, if it had enough water vapor to be in thermodynamic equilibrium with an adjoining flat surface of water. This too is a function of temperature and pressure. The functional relationship that defines the saturation mixing ratio is called the Clausius-Clapeyron Equation.

*Saturation Equivalent Potential Temperature:* The temperature a parcel of air at a given temperature and pressure would have if it were saturated, if all that water vapor were then condensed and removed, and if the air parcel were then brought down to some particular standard pressure. Since condensation gives off heat, condensation has a warming effect on the air (unlike evaporation, which absorbs heat and thus has a cooling effect). So an air parcel’s saturated equivalent potential temperature is always warmer than its potential temperature.

The reason all of these can be drawn on a sounding diagram is precisely because each is determined uniquely by a parcel’s temperature and pressure. In fact, if you didn’t have all those things drawn on your sounding diagram, you could draw them yourself by computing their values for several combinations of temperature and pressure and then drawing isopleths on the diagram. Frankly, though, you’ll never need to do that except perhaps in homework.

Except for Poisson’s Equation, you haven’t been told the specific functional relationships between temperature, pressure, and the other stuff. It should be obvious that whatever those relationships are will determine

what the resulting lines will look like. As it happens, all the patterns are fairly simple, and it is possible to distinguish the potential temperature, saturation mixing ratio, and saturation equivalent potential temperature at any point on a sounding diagram just from their relative orientations. Start with a line segment parallel a temperature line. If you pivot the line segment by rotating it counter-clockwise, soon it will be parallel to the lines of constant saturation mixing ratio. Rotate it some more, and it will be parallel to the lines of constant saturation equivalent potential temperature. Rotate it yet again, and it will become parallel to the lines of constant potential temperature. Eventually if you rotate it enough, it will become horizontal, which on a skew-T log-p diagram is parallel to the lines of constant pressure.



Now consider a point on a sounding diagram given by an air parcel's pressure and dew point. The definition of dew point is the temperature to which an air parcel would have to be cooled to become saturated, keeping pressure constant. So at a parcel's dew point temperature, the saturation mixing ratio corresponding to that point is the actual mixing ratio of the air parcel. Thus, by plotting both temperature and dew point (and looking at the sounding diagram), we can determine both the saturation mixing ratio and the actual mixing ratio, and thereby get the relative humidity.

Many other neat graphical meteorological tricks are also possible with a sounding diagram. Some, like estimating relative humidity, are

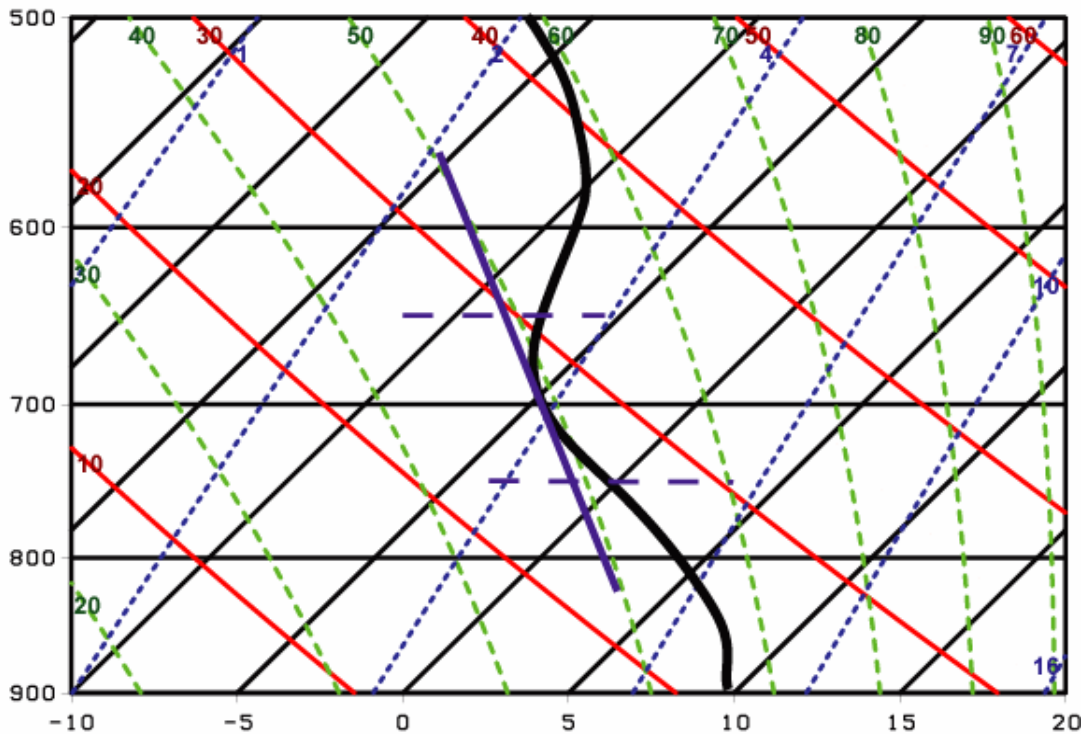
actually useful. Lots of them involve using the thermodynamic diagram aspects of a sounding plot to graphically determine various thermodynamic quantities. While the sounding, as I've described it, is a record of the vertical distribution of temperature (and anything else that can be computed from temperature and pressure), other uses of a sounding diagram often involve drawing lines that represent the evolution of temperature and pressure of a particular air parcel under the action of some thermodynamic process such as adiabatic expansion. So the graphs on a sounding diagram might be snapshots of atmospheric structure, or they might be time histories of real or imaginary air parcels undergoing real or imaginary changes.

### **5.5 Vertical Derivatives and the Hydrostatic Equation**

Vertical derivatives can be estimated from sounding diagrams in much the same way that horizontal derivatives can be estimated from ordinary graphs. Admittedly, it will be a little confusing at first, because the graph is 'sideways', and because pressure rather than height is being used as the vertical coordinate.

(Pressure is doing double duty on a sounding diagram. On one hand, it's a thermodynamic variable, part of the basic description of the characteristics of any air parcel. On the other, it serves as a vertical coordinate for mesoscale and larger (i.e., "weather-sized") atmospheric motions. The really confusing aspect of using pressure as a vertical coordinate is that pressure increases downward rather than upward. If you try to use pressure as the vertical part of a Cartesian three-dimensional coordinate system, you have to be careful because the coordinate system is no longer right-handed, complicating vector manipulation.)

With a normal graph you can quickly spot where the slopes (derivatives) are large and where they are small. With sounding diagrams, you'll have to forego your intuition for a little while and go back to the brute force technique of using two points along a line tangent to the graph of temperature vs. pressure. First, pick the pressure where you want to compute the derivative. Then, draw a line tangent to the sounding graph at that level. Then, look downward (because pressure increases downward), eyeball how much the temperature changes along your tangent line over some pressure change such as 50 mb, and divide by the magnitude of that pressure change. The same tangent line can be used to estimate the derivative with respect to pressure of potential temperature, saturation mixing ratio, etc.: instead of estimating the change in temperature along that line over some distance, estimate the change in potential temperature or whatever, over that distance.



What is the derivative of temperature with respect to the vertical coordinate, pressure, at 700 mb in this sounding? To begin, draw a line tangent to the sounding at 700 mb. The tangent line is solid blue in the figure. Now, consider levels 50 mb above and below 700 mb, shown by dashed lines. The value of temperature along the tangent line at 650 mb is about  $-8.5\text{ C}$ , and the value of temperature along the tangent line at 750 mb is about  $-1.5\text{ C}$ . So the change in temperature as pressure increases by 100 mb is  $(-1.5\text{ C}) - (-8.5\text{ C}) = 7\text{ C}$ . The derivative  $dT/dp$  is  $7\text{ C} / 100\text{ mb}$ , or  $0.07\text{ C}/\text{mb}$ .

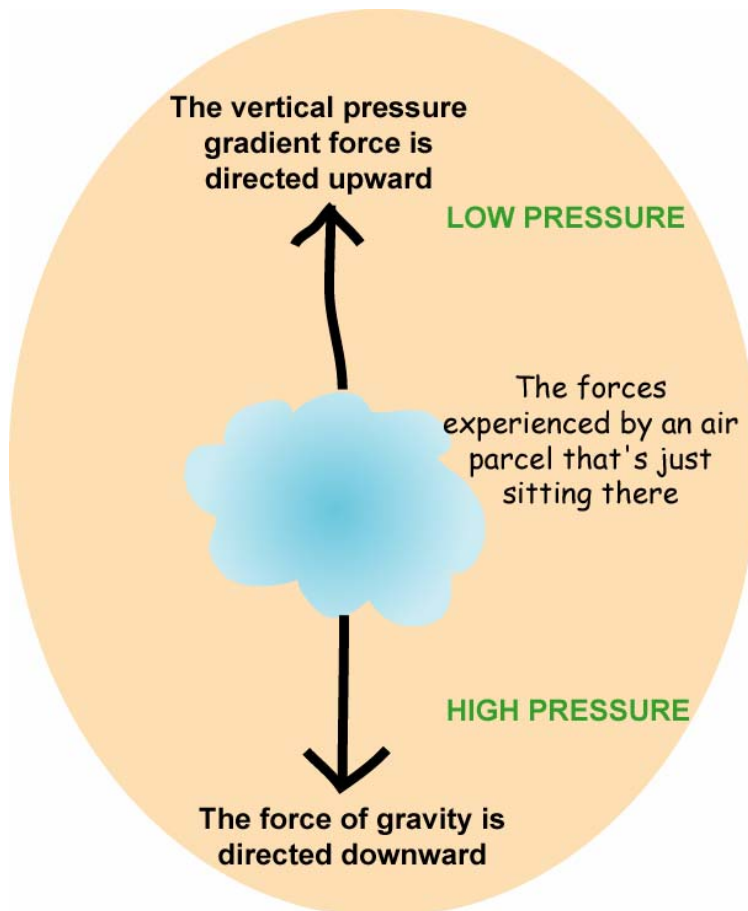
Derivatives of other quantities with respect to pressure can be estimated the same way. The change of potential temperature over the same interval is  $25\text{ C} - 29.5\text{ C} = -4.5\text{ C}$ , so the derivative  $dq/dp = -4.5\text{ C} / 100\text{ mb}$ , or  $-0.045\text{ C}/\text{mb}$ .

For most meteorological applications, though, what we really care about are the vertical derivatives with respect to height rather than with respect to pressure. So we need a handy way to convert from pressure derivatives to height derivatives. That handy way is the hydrostatic equation.

The hydrostatic equation is a distant cousin of the advection equation. Like the advection equation, it is a simplified version of the laws of physics or thermodynamics that describe how things evolve. In this particular case, we're dealing with vertical accelerations and

Newton's Second Law. Accelerations are caused by forces, and in the vertical direction there are two big forces to worry about: the force of gravity, and the force caused by the vertical pressure gradient.

If we take the simple, boring case of an atmosphere that's just sitting there, there are no accelerations going on and the two vertical forces must balance each other. Since gravity is always directed downwards, the vertical pressure gradient force must be directed upwards. Indeed it is: the vertical component of the pressure gradient force per unit volume of air is  $-dp/dz$ . This quantity is positive as long as pressure increases downwards. We can convert this into the resulting acceleration by dividing by density, the mass of air per unit volume:  $-1/\rho dp/dz$ . The other important acceleration would be gravity, which being a downward acceleration is negative:  $-g$ . If the air's just sitting there, these two accelerations must sum to zero:  $-1/\rho dp/dz - g = 0$ . Rearrange into its most common form:  $dp/dz = -\rho g$ .



While this is the most common form, it's not particularly useful for our present purposes. Sounding diagrams generally don't have density plotted. Sure, we could compute it using the ideal gas law, but it's better to just plug in the ideal gas law in its conventional meteorological form ( $p$

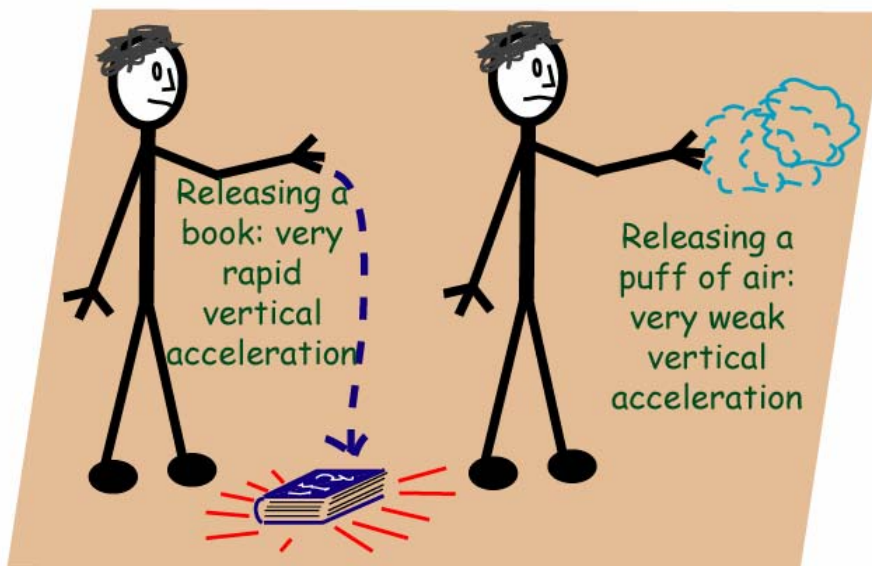
$= \rho R_d T$ ) to get :  $dp/dz = - p g / R_d T$ . We can now remove one variable by bringing pressure to the left hand side and utilizing the chain rule to get  $d \ln(p)/dz = - g / R_d T$ . In words, the rate of decrease of the natural log of pressure with height is inversely proportional to air temperature.

Before going on, a word about the ideal gas law. You thought you knew the ideal gas law as  $p V = n R^* T$ . The trouble with using that form of the ideal gas law directly on the atmosphere is that it's designed for an enclosed system, with an easily defined volume and number of molecules. To get something more useful, divide by  $V$  and then multiply the right-hand side by  $m_d/m_d$ , where  $m_d$  is the average molecular mass of dry air. That gives us

$$p = (n m_d/V) (R^*/m_d) T$$

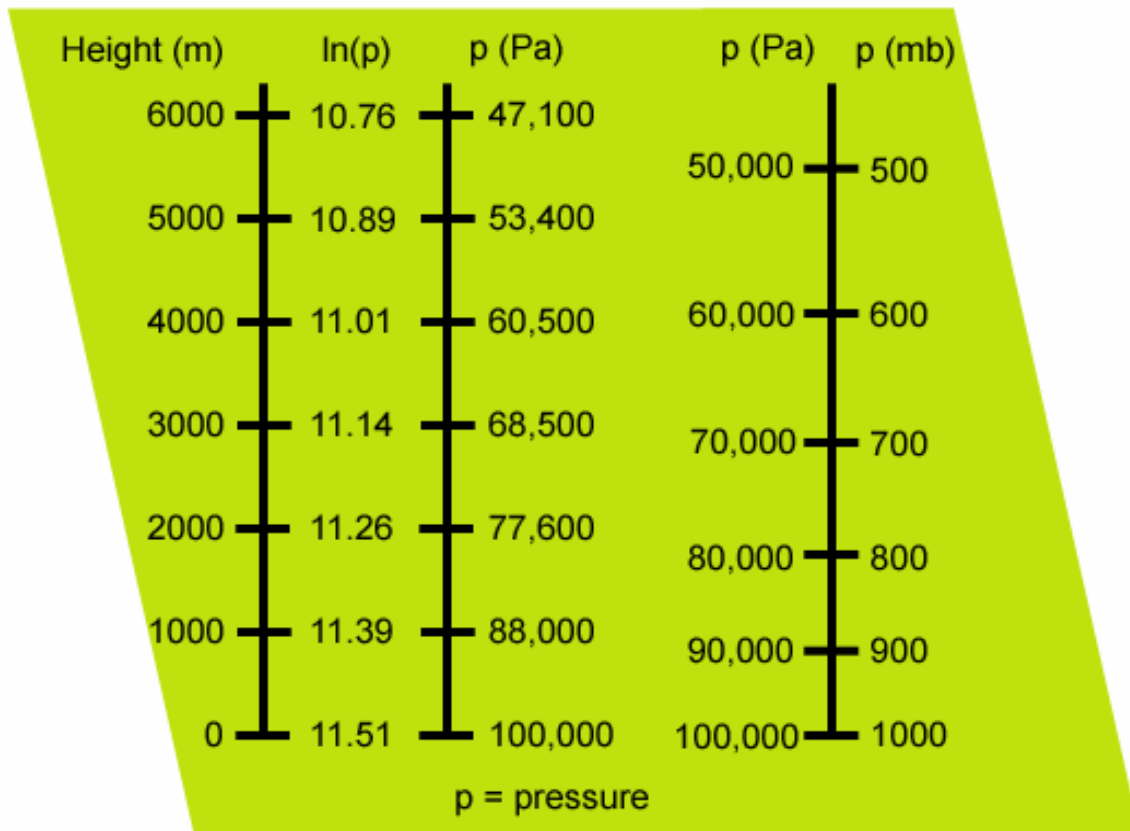
The first term in parentheses, the number of molecules times the mass per molecule divided by the volume, is just the mass divided by the volume, also known as the density. The second term in parentheses is a constant that doesn't depend on temperature, pressure, etc., only on the fact that we are dealing with air. So we can define  $R_d = R^*/m_d = 287$  J/kg/K. Thus we get the conventional atmospheric form of the ideal gas law.

We will, in later chapters, find lots of uses for the hydrostatic equation. But first, how good is the approximation that the two vertical forces balance? Think about how fast something accelerates under the influence of gravity, if you drop it. Typically, air does not accelerate nearly so fast (it would be quite a turbulent world if it did!). Ordinary



vertical accelerations are about one hundredth (two orders of magnitude less) of this value. So even when air really is accelerating vertically, the hydrostatic equation should still be 99% accurate.

One thing the hydrostatic equation tells us is that if a vertical segment of the atmosphere is isothermal, changes in height are directly proportional to changes in the log of pressure. In other words, if you go up 100 m, the log of pressure decreases by a particular amount. If you go up another 100 m, the log of pressure decreases by the same amount again. This is where the log-p part of the sounding diagram comes in. By



utilizing a vertical axis that's linear in log pressure rather than in pressure itself, the resulting sounding diagram is close to having a vertical axis that's linear in height. An inch anywhere on the sounding diagram corresponds to about the same vertical interval in the real atmosphere.

It would be nice to be able to have a sounding diagram that showed both pressure and height, but that's just not going to happen. Because the scaling between pressure and height depends on temperature, it is situation-dependent: each vertical profile of the atmosphere will have a different mapping of pressure and height. One thing that can be (and is)

done is to plot a height scale corresponding to some typical vertical distribution of temperature. Usually the so-called Standard Atmosphere is used. This has various attributes, including a surface (1000 mb) temperature of 15 C. In most parts of the United States, the actual surface temperature is usually within 20 degrees of that value, so the height scale will be accurate to within about 7%. (Where'd that number come from? When multiplying and dividing with temperature, always use Kelvins. And  $20\text{K}/288\text{K} = 6.9\%$ .)

## 5.6 Lapse Rates

The most common vertical derivative in meteorology by far is the lapse rate. The lapse rate is defined as the rate at which temperature decreases with height. Since  $\partial T/\partial z$  would be the rate at which temperature increases with height, the lapse rate is actually  $-\partial T/\partial z$ .

If all you have is temperature and pressure data, you can compute  $\partial T/\partial p$  and use the equations in the previous section to get  $-\partial T/\partial z$ , remembering to always work in Kelvins!

In addition to actual lapse rates, meteorologists also deal with hypothetical lapse rates. The most common of these is the dry adiabatic lapse rate. (Yeah, funky name, I know.) This is the lapse rate a layer of the atmosphere would have if that layer were dominated by turbulence that was continuously mixing the air. The value of that lapse rate is 9.8 C/km, at all ordinary temperatures and pressures.

We'll see in the next chapter that the well-mixed situation corresponds to potential temperature being constant with height, and therefore constant with pressure. So if a segment of a sounding has a temperature that exactly follows a line of constant potential temperature, that layer is either being continuously mixed or has recently been mixed. Furthermore, if you want to know the lapse rate of that layer, you need not compute it: it's 9.8 C/km.

For reasons discussed in the next chapter, if mixing is doing its job, any lapse rate greater than 9.8 C/km will be converted into 9.8 C/km by mixing. Thus, 9.8 C/km represents a common upper bound on the lapse rate, exceeded only when mixing is not vigorous enough. As you look at sounding diagrams, notice that the vertical profile of temperature is almost always moving toward higher values of potential temperature as one goes upward, meaning that the lapse rate is almost never greater than 9.8 C/km. Mixing usually does its job well.

## 5.7 The Buoyancy Equation

Section 5.5 introduced the hydrostatic equation. The hydrostatic equation expressed the balance between the force of gravity and the vertical pressure gradient force. This situation of balance accurately describes the typical state of the atmosphere, which is mostly just sitting there without accelerating upward or downward.

The interesting stuff happens when the atmosphere is not just sitting there. In particular, thunderstorms happen when there's some air that is not experiencing a nice, neat balance between gravity and the vertical pressure gradient force and instead accelerates upward.

Air pressure responds pretty quickly to imbalances. If you blow up a balloon, you've created a parcel of air inside the balloon that has a much higher pressure than the air outside. If you pop the balloon, the loud noise you hear is the sound wave triggered by the compression of air as the air inside the balloon suddenly rushes outward. Within a second or less, you can no longer detect any air motion or compressed air: the pressure has already equalized.

Now imagine that you make some air hotter than its surroundings. If there's an imbalance of the downward (gravity) and upward (pressure gradient) forces, the air should accelerate, either upward or downward depending on the direction of the imbalance. The so-called vertical momentum equation quantifies this:

$$\frac{Dw}{Dt} = -g - \frac{1}{\rho} \frac{\partial p}{\partial z}$$

Acceleration (in the vertical) is written  $Dw/Dt$ : the rate of change of the upward/downward wind speed ( $w$ ) in time. The use of capital D's rather than small d's or  $\partial$ 's indicates a special kind of derivative called the *total derivative*. It represents the rate of change of the value of a quantity associated with a particular air parcel. In this case,  $Dw/Dt$  is the rate of change of the vertical velocity of a particular air parcel, that is, the vertical acceleration. Since air parcels are imaginary, we'll take up the issue of how one computes a total derivative at a later date. For now, bear with me.

The vertical momentum equation states that vertical acceleration occurs when the accelerations attributable to the individual forces don't cancel each other, and that the rate of acceleration depends directly on the difference between the individual forces.

Does this equation make sense? Consider a solid object rather than a parcel of air. Since the density of the object is very large, the last term in the equation is very small and the acceleration due to gravity dominates. With the minus sign, gravity produces a negative acceleration, that is, an acceleration in the negative  $z$  direction. We say that the object starts to fall! Terminal velocity occurs when the object is moving so fast that it causes a pileup of air pressure underneath it, eventually large enough that the vertical air pressure variation balances the force of gravity so there's no more acceleration and the object falls at a constant rate.

**[Sidebar:** Unfortunately, the word “accelerate” means one thing in science and a different thing in common usage. In science, to “accelerate” means to “change speed or direction”. In common usage, to “accelerate” means to “speed up”. The common meaning of acceleration is just one type of scientific acceleration. When this book uses the word “accelerate”, it intends the scientific meaning. When this book wishes to refer to something speeding up in particular, it will say “speeding up”.]

Back to our locally hot air. What changes when we make the air hotter? The force of gravity doesn't change. The expansion of the hot air might cause a temporary blip in the pressure pattern, but overall the vertical pressure gradient is dominated by the effect of the surrounding air so the pressure gradient force doesn't change much. The only thing that really changes is the density of the heated air parcel. The density changes because pressure equalizes quickly, and according to the ideal gas law the only other possible permanent effect of an increase in temperature is a decrease in density.

Now while the vertical pressure gradient force stays the same, the force per unit mass is larger because the density of the air is smaller. (In the vertical momentum equation, the vertical pressure gradient force is divided by density.) With the acceleration due to the vertical pressure gradient force larger in magnitude than gravity, the air parcel accelerates upward. No big surprise there: anyone who's ever observed a candle would be able to predict the same thing. It's nice that the equation that describes this fact is one of the simplest in meteorology.

There are actually two equations describing the state of the atmosphere in and around our heated air parcel. The first describes the surroundings, which still have the same density (I'll write it as  $\rho_0$ ) and pressure, and aren't accelerating:

$$0 = -g - \frac{1}{\rho_0} \frac{\partial p}{\partial z}$$

and the air parcel, which has a different density (I'll write it as a perturbation to the original density:  $\rho_0 + \rho'$ ):

$$\frac{Dw}{Dt} = -g - \frac{1}{\rho_0 + \rho'} \frac{\partial p}{\partial z}$$

Now, look what happens when we subtract the top equation from the bottom one:

$$\frac{Dw}{Dt} = \frac{1}{\rho_0} \frac{\partial p}{\partial z} - \frac{1}{\rho_0 + \rho'} \frac{\partial p}{\partial z}$$

$$\frac{Dw}{Dt} = \frac{\partial p}{\partial z} \left( \frac{1}{\rho_0} - \frac{1}{\rho_0 + \rho'} \right)$$

$$\frac{Dw}{Dt} = \frac{\partial p}{\partial z} \left[ \frac{(\rho_0 + \rho') - \rho_0}{\rho_0(\rho_0 + \rho')} \right]$$

$$\frac{Dw}{Dt} = \frac{\partial p}{\partial z} \left[ \frac{\rho'}{\rho_0(\rho_0 + \rho')} \right]$$

$$\frac{Dw}{Dt} = \frac{1}{\rho_0} \frac{\partial p}{\partial z} \frac{\rho'}{(\rho_0 + \rho')}$$

and, using the first equation again,

$$\frac{Dw}{Dt} = -g \frac{\rho'}{(\rho_0 + \rho')}$$

So if you have a negative density perturbation, which corresponds to a hot parcel, the right-hand side of the final equation will be positive and the acceleration will indeed be upwards. Interpreting this equation, it's as though there's a piece of the gravitational force that's not balanced by the vertical pressure gradient force, and the fractional amount of that imbalance depends on the fractional difference in density.

This is the buoyancy equation. We say that a parcel that is less dense than the surrounding air is positively bouyant and would accelerate upward. The buoyancy equation states mathematically the magnitude of that acceleration.

Just so you know, there is an important effect that's ignored by the buoyancy equation. It's the same effect responsible for terminal velocity: if a bunch of air starts moving upward quickly, it will perturb the pressure above and below it and change the vertical pressure gradient force. Fortunately, this effect is small enough to be safely ignored until graduate school.

One more form: an alternate form using potential temperature instead of density. It's not much different, if the noughts and primes have the same meaning:

$$\frac{Dw}{Dt} = g \frac{\theta'}{\theta_0}$$

The sign is different because a warm air parcel would have a higher potential temperature, just as it has a lower density.

### 5.8 The Thermodynamic Equation

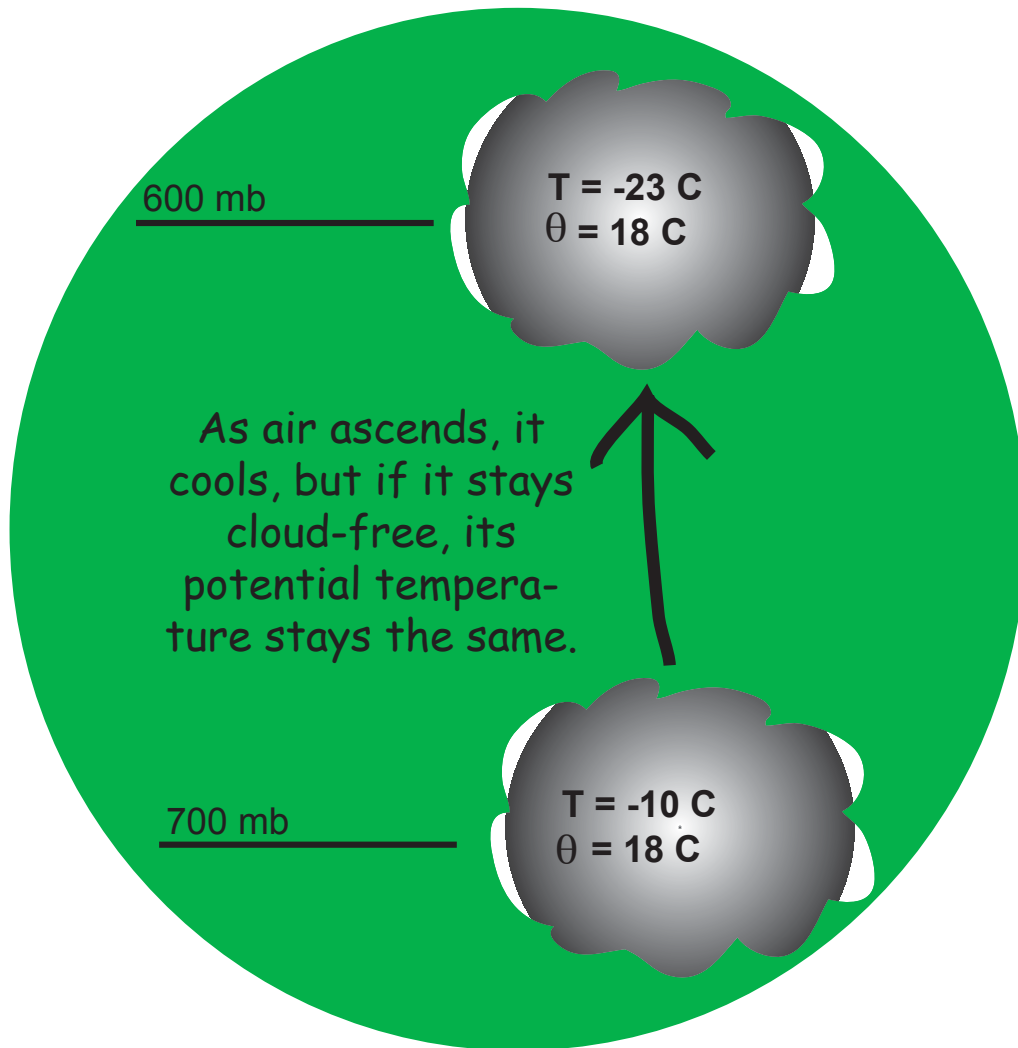
Just saying that an air parcel magically becomes warmer is one thing, but can it actually happen? To answer this question, we turn to the thermodynamic equation.

There are two basic versions of the thermodynamic equation. One describes the change in *temperature* of an air parcel due to various processes. The other describes the change in *potential temperature* of an air parcel due to various processes. The equation written in terms of potential temperature is inherently much simpler for a very plain reason: the most common process of the bunch is the one that affects temperature and not potential temperature: adiabatic expansion and contraction due to vertical motion. So the temperature form has one more term in it, one that cannot be neglected. But it can be sidestepped just by thinking in terms of potential temperature.

While rising air parcels cool and descending air parcels warm, the potential temperature of an air parcel is not directly affected by ascent or descent. To see this, remember the definition of potential temperature: the temperature an air parcel would have if brought down (or up) to 1000 mb without exchanging heat with its surroundings. Now suppose the parcel's halfway to 1000 mb. Has its potential temperature changed? No, it's still on the way to the same temperature at 1000 mb. No matter how the parcel ascends or descends, as long as it doesn't exchange heat with its surroundings it will end up at the same temperature if it ever arrives at 1000 mb. Thus, it always has the same potential temperature.

The equation governing the potential temperature of an air parcel is simply:

$$\frac{D\theta}{Dt} = \frac{\theta}{T} \frac{Q}{c_p}$$

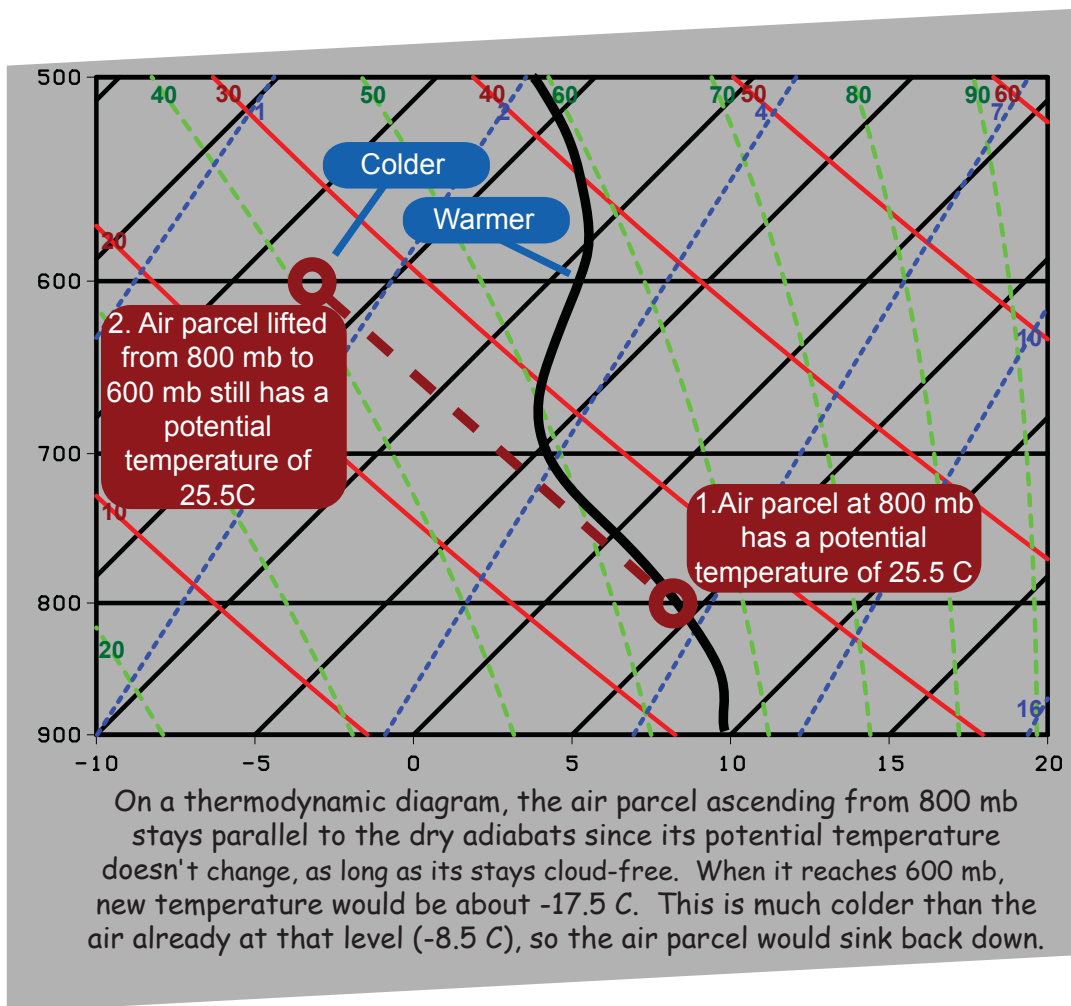


The mysterious-looking  $Q$  is a catchall variable that represents the addition (or removal, if negative) of heat due to phase changes, heat exchange, etc. The  $c_p$  is the heat capacity of air at constant pressure per unit mass. If there's nothing going on with  $Q$ , the right hand side will be zero and potential temperature won't change.

Now imagine you're following an air parcel that is rising and falling, but not exchanging heat. Its potential temperature is constant, so no matter what pressure or temperature that air parcel achieves, the points on the sounding diagram that represent the parcel's temperature and pressure must always be along the same line of constant potential temperature. For reasons related to the thermodynamic term 'adiabatic', lines of constant potential temperature on a sounding diagram are referred

to by the romantic name ‘dry adiabats’. Thus, we say that an ascending or descending air parcel “follows a dry adiabat” on a sounding diagram.

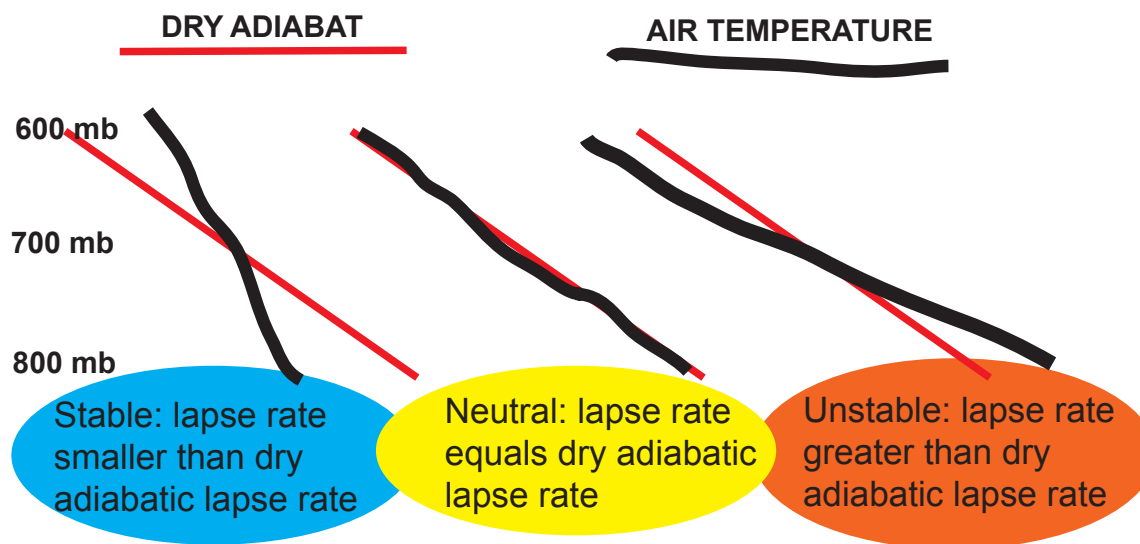
We can use that fact, combined with the buoyancy equation and any atmospheric sounding, to ask “What if?” questions. What if air at 800 mb were to ascend to 600 mb? Would it be positively buoyant, negatively buoyant, or neutral? To answer that question, follow the air parcel’s dry adiabat on the sounding diagram up to 600 mb, and compare the parcel’s temperature at that level with the temperature of the air already at that level. If the parcel is warmer, it will be less dense and positively buoyant and will rise (accelerate upward). If the parcel is cooler, which is the normal course of events, it will be more dense and will sink (accelerate downward).



We can answer the “What if?” question in an even simpler way. We know that the air parcel, when it reaches 600 mb, will have the same potential temperature as when it started. So simply read off the parcel’s potential temperature and compare it to the potential temperature of the air

that's at 600 mb already. The buoyancy equation will tell you whether that air parcel will rise or sink, and exactly how rapidly it would do so.

A little terminology will close out this section. If you imagine lifting an air parcel a little ways, as might happen in the real atmosphere due to random turbulence, its vertical excursion would be hindered if it becomes negatively buoyant and helped if it becomes positively buoyant. In the former case, it would accelerate back downward toward where it started, while in the latter case it would flee upward. We call the first case 'stable', since motion is inhibited, and the second case 'unstable', since motion is enhanced. The in-between situation, when the lifted parcel exactly matches its surroundings and doesn't accelerate at all, is 'neutral'.



### 5.9 Latent Heat Release

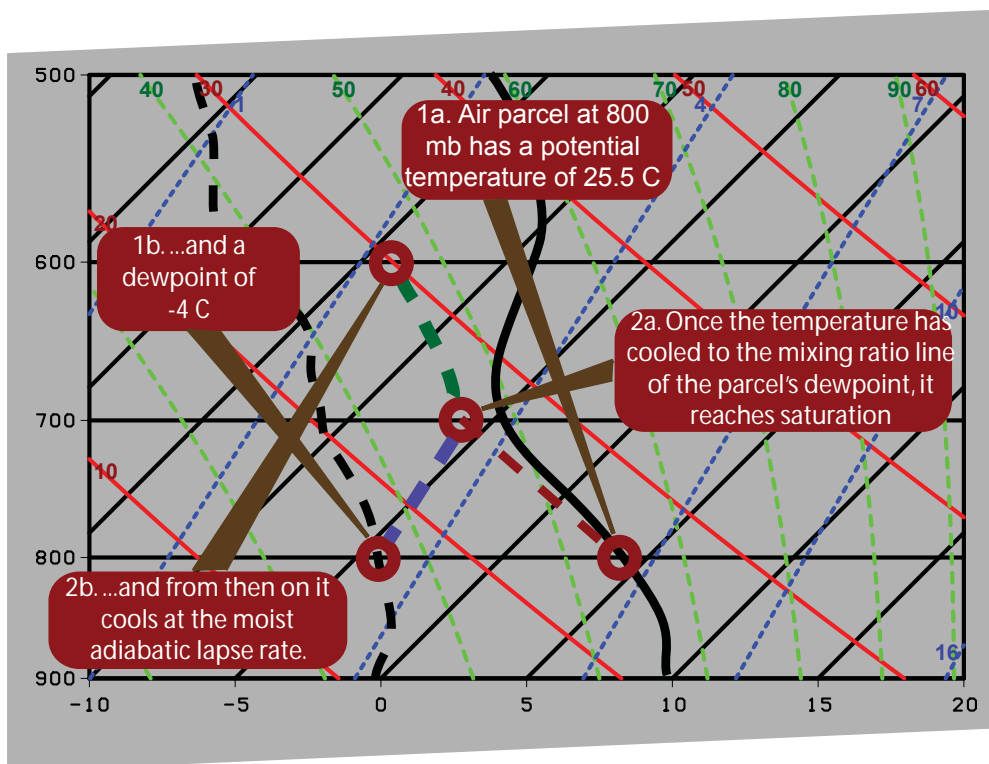
The term 'unstable' implies some serious consequences. Indeed, any such instability rapidly generates turbulence, and without some source of additional instability, the original instability goes away. Thus, most of the time, almost all levels in the atmosphere are stable to dry vertical displacements.

But vertical motion can sometimes provide its own heat, turning a stable situation into an unstable one. That source of heat is condensation, and the process is known as latent heat release. The idea behind that name is that potential heat energy is stored in the gas phase of water vapor when water evaporates, and when the water condenses again, that energy is released as heat.

In a perversely paradoxical situation, the most common form of latent heat release only occurs while the temperature of an air parcel is actually going down. So, even though condensation is heating the air parcel, its temperature is dropping. Why? You need the dropping temperature, caused by an air parcel ascending and expanding, to cool the air to saturation and initiate condensation. The more the air cools, the greater the amount of condensation that must take place to keep the air from exceeding 100% relative humidity much. The trick is, even though the air parcel is cooling, the fact of condensation makes the air parcel cool less rapidly than it would otherwise. Actually, it cools following the moist adiabatic lapse rate when it's condensing, and the dry adiabatic lapse rate when it isn't.

The effect of condensation is much simpler in potential-temperature-land. A rising air parcel stays at the same potential temperature as long as no condensation takes place. But once condensation does occur, the potential temperature of the air parcel goes up. It's the thermodynamic equation in action!

So now go back to the hypothetical air parcel rising from 800 mb to 600 mb. If it hits saturation at 700 mb, it's going to warmer than the original example by the time it reaches 600 mb, because a lot of water vapor will have condensed. Now maybe the air parcel that was stable before, now because of condensation is warmer than the 600 mb surroundings and is unstable.



How do you tell if and when an ascending air parcel is going to become saturated? That's where the saturation mixing ratio lines come in. As air ascends, the proportion of the air parcel made up of water vapor (that is, the mixing ratio) stays constant. As the air parcel cools, its saturation mixing ratio decreases. When the saturation mixing ratio drops so low that it equals the mixing ratio itself, the parcel is saturated. Beyond that level, the air cools at the moist adiabatic lapse rate.

### Questions

1. Obtain a printout of a sounding and plot it up on a sounding diagram.
2. Obtain an upper air map and decode the observations from five stations.
3. Obtain a sounding diagram and write down the data values at each level.
4. Obtain a sounding diagram and estimate the derivatives with respect to pressure and height of temperature and potential temperature at the 510 mb level.
5. Using the potential temperature lines on a sounding diagram, verify the dry adiabatic lapse rate of  $9.8 \text{ C/km}$ .
6. For current values of temperature and pressure, use the moist adiabats on a sounding diagram to estimate the moist adiabatic lapse rate. Describe the temperature and pressure conditions under which the dry and moist adiabatic lapse rates are most similar.
7. Take a sounding and pick an air parcel. Lift it along a dry adiabat for 150 mb. Assume it becomes saturated at that point. Then lift it along a moist adiabat for 150 mb. Report on the following quantities: (a) the original temperature of the air parcel, (b) the temperature of the air parcel 150 mb higher, (c) the temperature of the air parcel 300 mb higher, and (d) the temperature of the air parcel 300 mb higher if it had never become saturated. Does the air parcel cool as it rises whether or not it is saturated?