Do the Local Motion. Part II: Go with the Flow
(Atmo 336, Fall 2007)

Getting Started. The simple stretching and shearing concepts considered at the end of Part I are often not the most intuitive ways to describe a flow field. Instead, it's often better to *group* the stretching and shearing terms to produce quantities with more direct physical interpretations.

The present lab builds on the results of Part I by using the same three scripts as before: *particle.m*, *velocity.m* and *flowmovie.m*. But instead of simple stretching and shrearing flows our attention now turns to:

$$\text{the divergence} \quad \nabla \cdot \mathbf{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$
$$\text{the vorticity} \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$
$$\text{the strecthing deformation} \quad T = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$
$$\text{and the shearing defomation} \quad S = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

The divergence. First a divergent flow. Reset the velocity to be

$$u = x \quad , \quad v = y \tag{1}$$

and create the animation for the square. Save the movie to the file *expansion.avi* and copy to the figure directory to turn in.

(a) What are the vorticity, divergence and two deformations for this flow?

(b) In addition to creating the movie, the *flowmovie* command also stores the time-dependent positions for each of the N points along the square (refer to the figure from Part I). These positions are stored in the 2D arrays $x(j,k)$ and $y(j,k)$, where the first index refers to the number of the point (as shown in the previous figures) while the second indicates the time. Note that the time is measured in units of dt, so that a time index of k actually refers to a time of $t = (k\text{-}1)\,dt$. So for instance, the variable $x(j,k)$ refers to the x-coordinate position for the j^{th} point along the square at the time $t = (k\text{-}1)\,dt$ [and similarly for $y(j,k)$].

Using the positions of the corner points on the square, compute (and plot) the area of the square as a function of time. (You should probably put this in an m-file, in case you need to modify it later.) Your code should look something like

```
for k=1:nt
   t(k) = (k-1).*dt;
   delx = ;
   dely = ;
   area(k) = ;
end
```

```
plot(t,area,'k-','linewidth',2);
xlabel('time','fontsize',16);
ylabel('particle area','fontsize',16);
set(gca,'xtick',[0 0.2 0.4 0.6 0.8 1.0]);
set(gca,'ytick',[0 5 10 15 20 25 30]);
hold on;
```

(c) We showed in class that the area A of an expanding particle is described by

$$\frac{1}{A}\frac{dA}{dt} = \boldsymbol{\nabla} \cdot \mathbf{u}$$

where $A = \Delta x\, \Delta y$. Solve for A theoretically (i.e., with pencil rather than computer) using the velocity definition in (1) along with the initial area for the particle. Overlay your result on the previous plot using something like

```
for k=1:nt
   area_th(k) = some function of t(k);
end
plot(t,area_th,'r–','linewidth',2);
hold off;
```

Does the theoretical prediction agree with the animated result? Save your figure to the file *area_div.jpg* and copy to the figure directory to turn in.

The vorticity. Now for a vortical (or rotational) flow. Reset the velocity to be

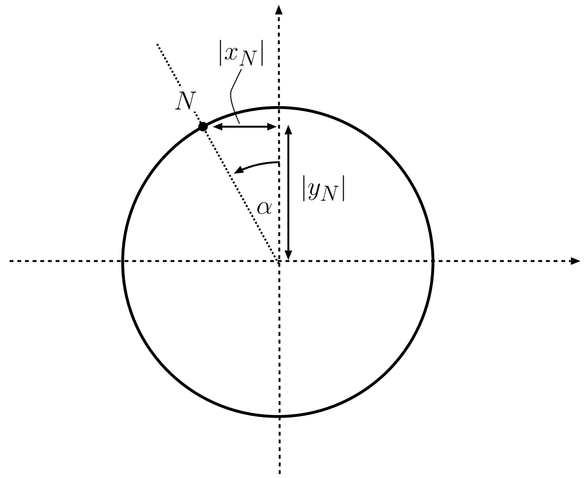
$$u = -y \quad , \quad v = x$$

but this time create the animation for the circle rather than the square. Save the animation to the file *rotation.avi* and copy to the figure directory to turn in.

(d) What are the vorticity, divergence and two deformations for this flow?

(e) Note that if you've done things correctly, the point on the circle labeled N initially starts out along the y-axis. (Refer to your particle definition for the circle—as well as the figure from the previous lab—and modify as necessary.) Suppose that the rotation angle of this point (as measured in radians) at the end of the animation is given by α (as illustrated at right). We can then estimate the rotation rate (or angular frequency) for the rotation using

$$\alpha = \omega t \quad \text{or} \quad \omega = \frac{\alpha}{t}$$



Compute ω from your position data and record your result. Your code should look something like

```
t_nt = (nt-1).*dt;
xN = abs(x(N,nt));
yN = abs(y(N,nt));
alpha = ;
omega = ;
```

where the *abs* command refers to the absolute value.

(f) In class we showed that the vorticity ζ is related to the angular frequency ω of the motion by

$$\zeta = 2\,\omega \quad \text{so that} \quad \omega = \frac{\zeta}{2}$$

Does this theoretical prediction agree with the animated result?

The stretching deformation. Next a pure stretching deformation. Reset the velocity field to

$$u = x \quad , \quad v = -y$$

and create the animation for the square. Save your movie to the file *stretch_def.avi* and copy to the figure directory to turn in.

(g) Compute the divergence, vorticity, and stretching and shearing deformations for this flow.

(h) We argued (well, ok, I argued) in class that a deformation flow produces a change in the shape of a small fluid particle but does not produce a change in the net particle area. Verify that this is true by computing Δx, Δy and the area $\Delta x\,\Delta y$ for the square. Your code should look something like:

```
for k=1:nt
  t(k) = (k-1).*dt;
  delx(k) = ;
  dely(k) = ;
  area(k) = ;
end
plot(t,delx,'k-','linewidth',2);
hold on;
plot(t,dely,'b-','linewidth',2);
plot(t,area,'r-','linewidth',2);
```

```
xlabel('time','fontsize',16);
ylabel('dx, dy, area','fontsize',16);
set(gca,'xtick',[0 0.2 0.4 0.6 0.8 1.0]);
set(gca,'ytick',[0 1 2 3 4 5 6]);
```

Save your figure to the file *area_def.jpg* and copy to the figure directory to turn in.

(i) How big is the net change in area compared to the initial area (expressed as a percentage)?

The shearing deformation. And at long last, a shearing deformation. This time we'll skip whole the song and dance and just do the movie. Reset the velocity to

$$u = y \quad , \quad v = x$$

and create the animation for the great state of Texas. Save to the movie to *shear_def.avi* and copy to the figure directory to turn in.

Figure and Movie Checklist. Take a look at your figure directory. You should have turned in the following:

- *Four movies:* expansion.avi; rotation.avi; stretch_def.avi; shear_def.avi
- *Two figures:* area_div.jpg; area_def.jpg

Take a minute to admire your work. Turn to the person at your left and take a look at their figures. Argue over whose figures are best.