Atmospheric Sciences 336, Fall 2007
Problem Set 4
Due Friday, Oct 26

Problem 1 *Vorticity + Convergence = ?*
Suppose that the kinematic properties of a 2D flow field are given as

$$D = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -B \qquad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = B$$
$$T = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \qquad S = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 0$$

where B is a positive constant.

(a) Assuming a reference frame in which the fluid is stationary at the origin, solve for the velocity (u, v) as a function of x and y.

(b) Sketch the velocity field derived in (a).

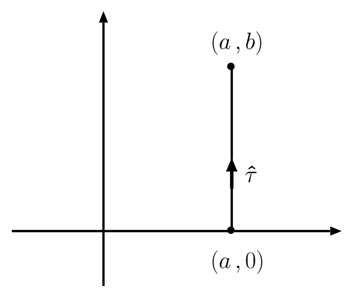
(c) Consider a small rectangular particle centered at the origin that moves with the flow. Would you expect this rectangle to be expanding? Rotating? Changing shape? Explain.

Problem 2 *Off the beaten path...integral*

(a) Compute the path integral of the function

$$f(x, y) = xy$$

along the straightline path starting at $(a, 0)$ and ending at (a, b).



(b) Now suppose that the function is given by

$$f(x, y) = \mathbf{u} \cdot \hat{\boldsymbol{\tau}} \quad \text{where} \quad \mathbf{u} = (x, y)$$

and where $\hat{\boldsymbol{\tau}}$ is the unit vector tangent to the path (typically referred to as the *unit tangent*). Compute the path integral along the same path as that described in (a).

Problem 3 *Vorticity is to circulation as divergence is to...*
Consider the two flow fields defined by

$$(u, v) = \frac{A}{2}(x, y) \qquad \text{and} \qquad (u, v) = \frac{B}{2}(-y, x)$$

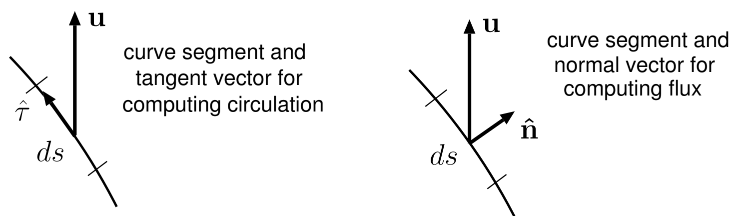
where A and B are constants.

(a) Compute the vorticity and divergence for each flow.

(b) Recall that the circulation is defined to be the path integral around a closed curve of the tangential wind component; i.e.,

$$C = \oint \mathbf{u} \cdot \hat{\boldsymbol{\tau}} \, ds$$

where ds is the increment of distance along the path and where $\hat{\boldsymbol{\tau}}$ is the unit tangent to the path (as illustrated below.) For each of the two flows in (a), compute the circulation around the rectangular path with corner points $(-L, -L/2)$, $(L, -L/2)$, $(L, L/2)$ and $(-L, L/2)$. How is the circulation for this curve related to the vorticity?



(c) Analogous to the circulation, the *flux* across a closed curve is defined as the path integral around the curve of the normal (or perpendicular) component of the wind; i.e.,

$$F = \oint \mathbf{u} \cdot \hat{\mathbf{n}} \, ds$$

where now $\hat{\mathbf{n}}$ is a unit vector in the direction perpendicular to the curve (as illustrated above). By convention, we choose $\hat{\mathbf{n}}$ to point in the direction away from the interior of the curve. For each of the two flows defined in (a), compute the flux across the rectangle defined in (b). How is the flux related to the divergence?

(d) Pretend you're taking the SAT again and finish the analogy started (in italics) at the beginning of the problem.

Problem 4 *Streamfunctions and streamlines*

Consider the streamfunction defined by

$$\psi = -Uy + \frac{U}{k}\sin(kx)$$

where U and k are positive constants. Find the velocity vector (u, v) and sketch the associated streamline pattern. (*Hint:* pick a particular streamline and solve for y as a function of x. Then pick another streamline and do the same. Rinse and repeat.)