Atmospheric Sciences 336, Fall 2007
Problem Set 1
Due Wednesday, Sept 12

Problem 1 *The Argentinian Zonda*
The next big observational program in the field of mountain meteorology will be held roughly two years from now in the South American Andes. So we better start getting ready now. Consider the following:

The term *Zonda* refers to a warm dry westerly wind that occurs along the lee slope of the Argentinian Andes. These winds typically occur when a relatively cold and dense low-level air mass on the upstream side of the Andes is blocked and thus not able to pass over the mountain peak. This blocking of the low-level air then causes air from aloft to descend along the lee slope—much like water spilling over the top of a dam. The warm dry nature of the wind is caused by the adiabatic warming (and associated drying) of this air as it descends.

Suppose that a Zonda wind downstream of the Andes is measured to have a temperature of 298 K at a surface pressure of 950 hPa. Assuming that the air in the wind originated at roughly 600 hPa pressure (roughly a height of 4 km), calculate the temperature of this air before it began its descent. (Assume that the descent is approximately adiabatic.)

Problem 2 *Go ahead, try it*
Using the first law of thermodynamics, explain why the air let out of a car tire is generally cooler than the air around it. (Assume that the tire is at the ambient air temperature—that is, you haven't driven in a while.)

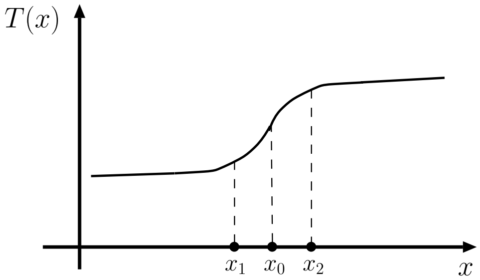
Problem 3 *The concrete jungle*
Consider a 1 kg mass of air in contact with black asphalt at the ground. Over some time period in the late morning the asphalt adds exactly 1000 J of heat to this air mass without changing the pressure. How much of this heat goes into changing the internal energy of the mass and how much goes to the work done by the mass as it expands?

More on the back!

Problem 4 *More Taylor series.* Suppose that the temperature distribution across a front is as illustrated below, where x measures the distance normal to the front. Measurements of the temperature are recorded at three locations: x_0, x_1, and x_2, with x_0 at the center of the front as illustrated in the figure. The three locations are equally spaced so that

$$x_2 - x_0 \;=\; x_0 - x_1 \;=\; \Delta x$$

where Δx is a known separation distance.



(a) By definition, the average slope of the function $T(x)$ between x_1 and x_2 is given by

$$\textit{Slope} \;=\; \frac{\textit{Rise}}{\textit{Run}} \;=\; \frac{T(x_2) - T(x_1)}{2\Delta x} \tag{1}$$

Show that this average slope can be used to approximate the temperature *gradient* dT/dx seen at the center point x_0. Specifically, show that

$$\left.\frac{dT}{dx}\right|_{x_0} = \frac{T(x_2) - T(x_1)}{2\Delta x} - \frac{1}{6}\left.\frac{d^3T}{dx^3}\right|_{x_0}(\Delta x)^2 + \cdots$$

where the dots indicate terms of higher order in Δx. Typically we would write this as

$$\left.\frac{dT}{dx}\right|_{x_0} = \frac{T(x_2) - T(x_1)}{2\Delta x} + O((\Delta x)^2) \tag{2}$$

where the notation $O((\Delta x)^2)$ indicates an error term of order $(\Delta x)^2$ or higher.

(b) To be concrete, suppose that the temperature distribution near x_0 is given by

$$T(x) = T_s + T_0 \sin(x - x_0)$$

where T_s and T_0 are constants. How good would our average slope approximation in (2) be if Δx were $\pi/4$? Express your answer in terms of the normalized error measure

$$\textit{Err} = \frac{\textit{Slope} - dT/dx|_{x_0}}{dT/dx|_{x_0}} \tag{3}$$

where *Slope* is the average slope defined by (1).

(c) Now suppose that $\Delta x = \pi/8$. How good is the average slope approximation in this case [again in terms of (3)]?

(d) If Δx were to approach zero, what do you think would happen to the error? Is this consistent with (2)?